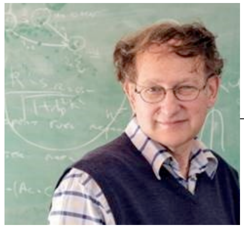
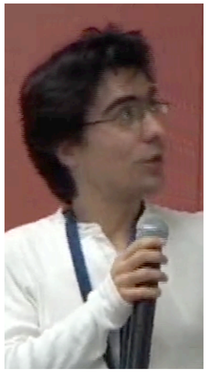
Nat Fisch-65, Princeton, 28 March 2016



Solved and Unsolved Dynamos

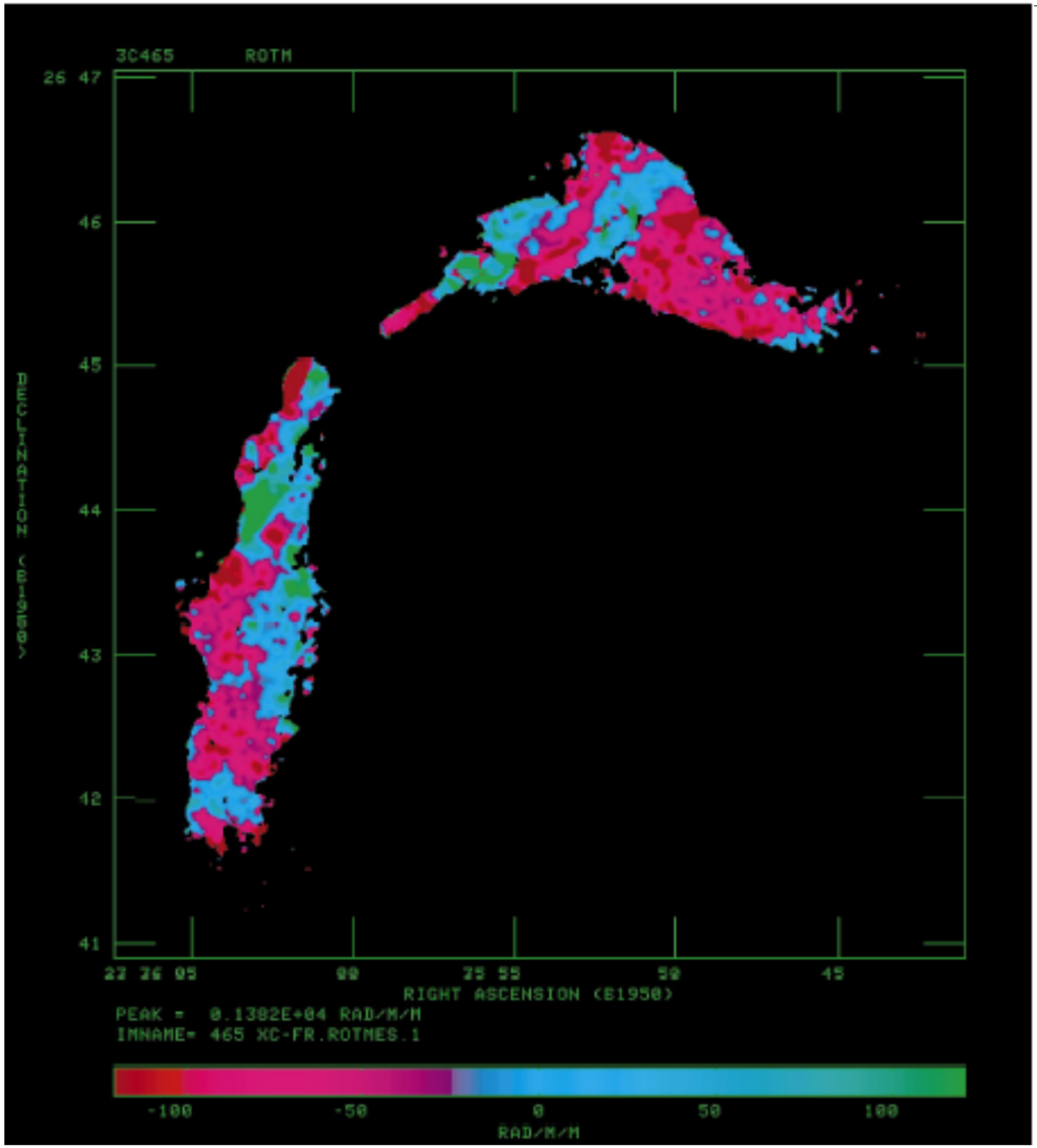
Alexander Schekochihin (Oxford)

S. Cowley (UKAEA), R. Kulsrud (Princeton),
G. Gregori, A. Bott, A. Rigby, J. Meinecke, T. White (Oxford),
P. Tzeferacos, C. Graziani, D. Lamb (Chicago),
F. Rincon (Toulouse), F. Califano (Pisa), F. Valentini (Calabria),
M. Kunz, J. Stone (Princeton), S. Melville (Harvard),
P. Helander (Greifswald), M. Strumik (Oxford)



Rincon, Califano, AAS, Valentini, *PNAS* in press (2016) [arXiv:1512.06455]
Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]
Rincon, AAS, Cowley, *MNRAS* **447**, L45 (2015)
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)
AAS et al., *PRL* **100**, 184501 (2008); Rosin et al., *MNRAS* **413**, 7 (2011)
Tzeferacos et al., in preparation (2016); Meinecke et al., *PNAS* **112**, 8211 (2015)
AAS et al., *NJP* **9**, 300 (2007); AAS et al., *ApJ* **612**, 276 (2004)

Cosmic Magnetism: Galaxy Clusters



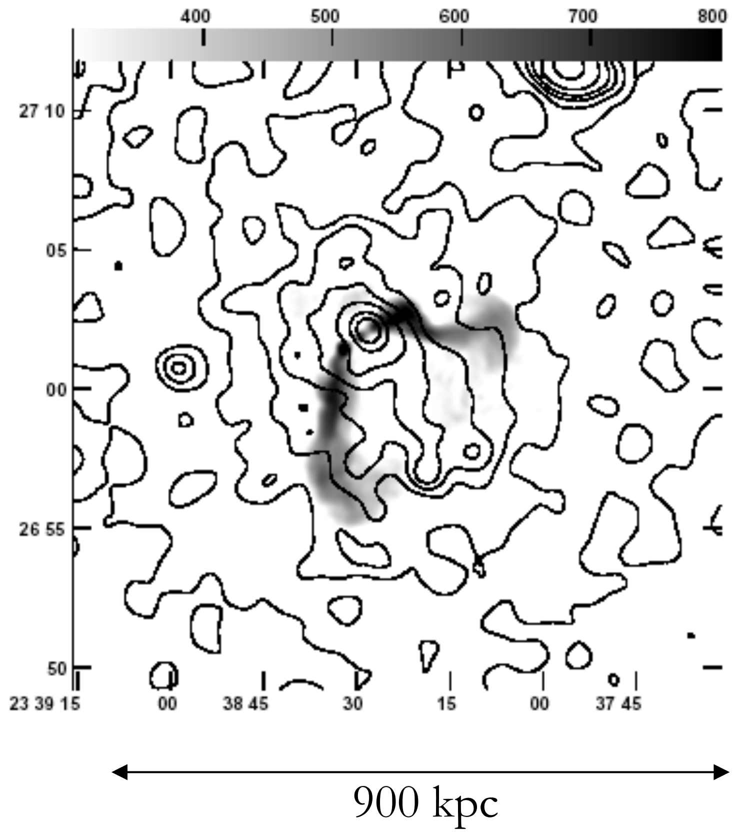
[**Abell 2634 cluster**, Eilek & Owen 2002, *ApJ* **5267**, 202]

Typically,

$$B \sim 10^{-6}\ \mathrm{G}, \quad \beta \sim 10^2$$

Crucially (imho),

$$\frac{B^2}{8\pi} \sim \frac{\rho u^2}{2}$$



Turbulence Makes the Field

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

This equation is linear in **B**, so field will either decay to zero or grow to dynamical strength. Probably the latter.

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Theoretical MHD Dynamo: ~SOLVED

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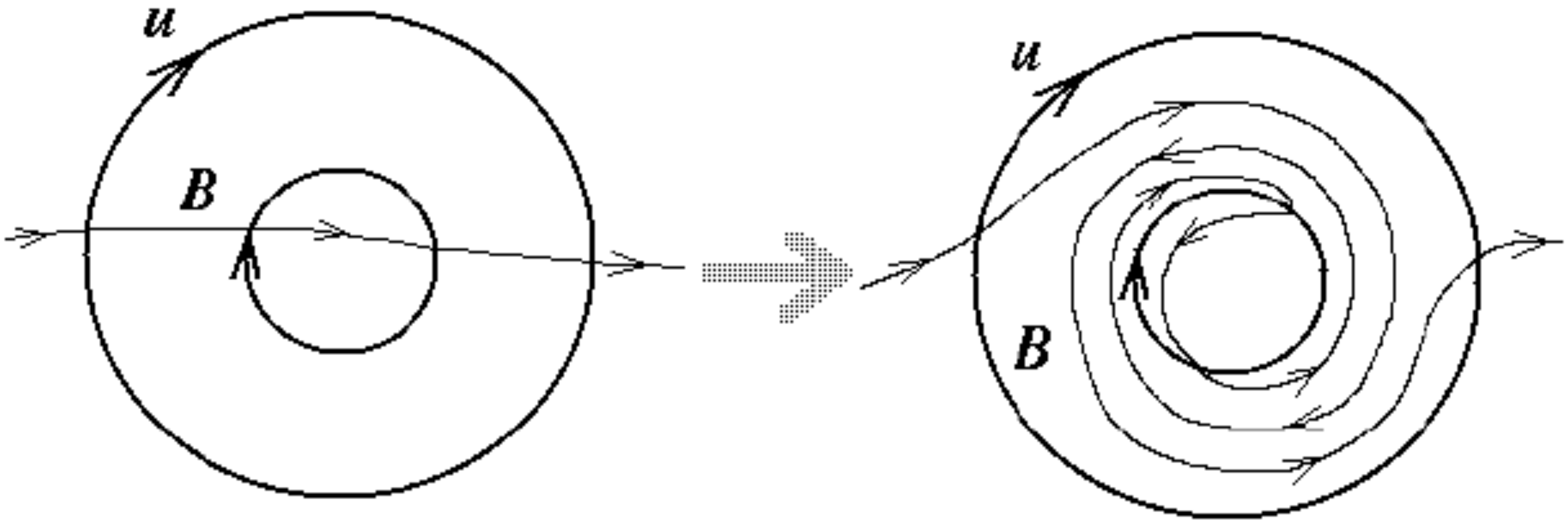

Straight field

Folded field

Theoretical MHD Dynamo: ~SOLVED

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \cancel{\eta \nabla^2 \mathbf{B}}$$

All you need is a
chaotic flow

Basic idea:

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \equiv \frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u}$$

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla \mathbf{u}) B \equiv \gamma B$$

$$\ln B \sim \int^t \mathrm{d}t' \, (\mathbf{bb} : \nabla \mathbf{u})(t')$$

So, roughly, field in Lagrangian frame accumulates as random walk

AAS et al., *ApJ* **612**, 276 (2004)

Theoretical MHD Dynamo: ~SOLVED

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \cancel{\eta \nabla^2 \mathbf{B}}$$

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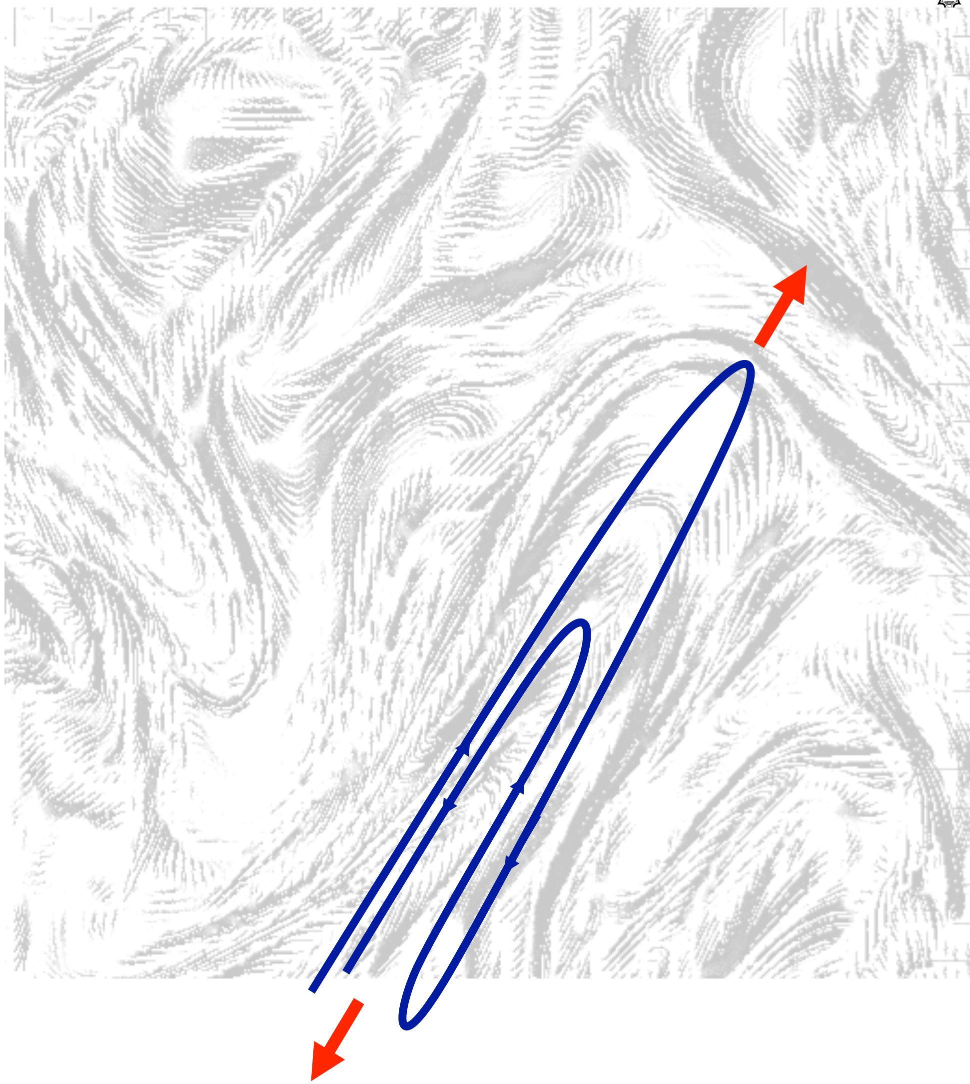

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Key effect: a succession of random stretchings (and un-stretchings)

AAS et al., *ApJ* **612**, 276 (2004)

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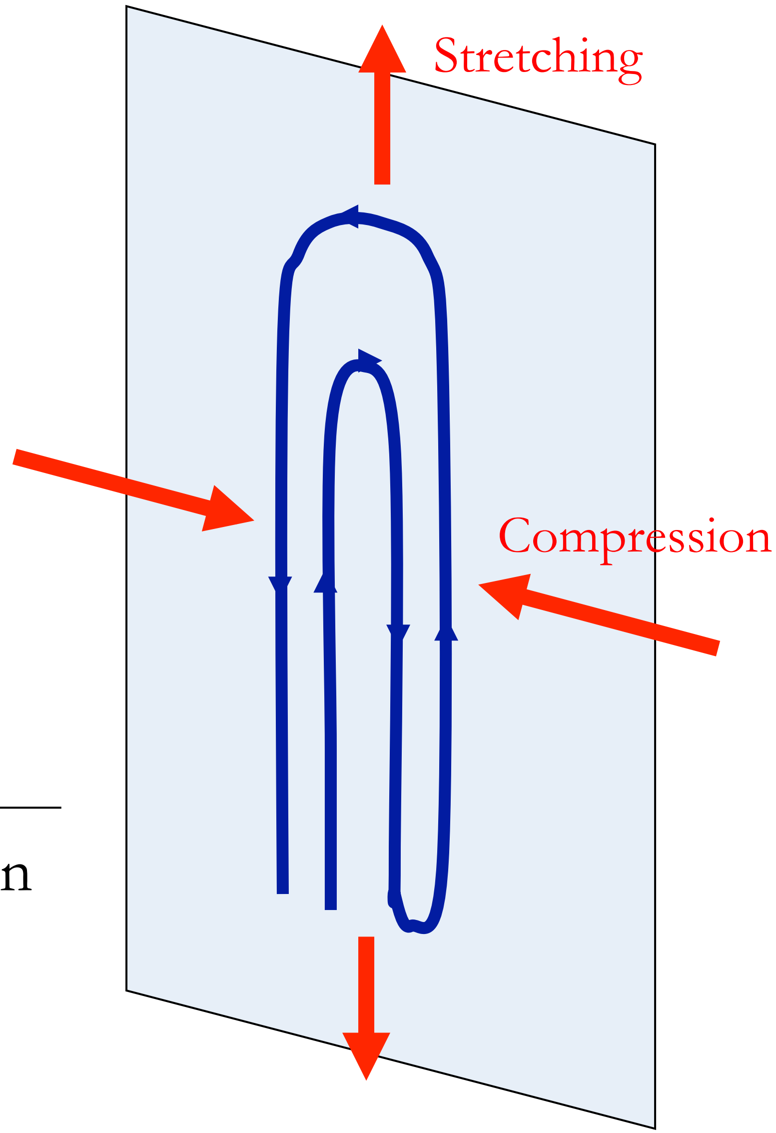
Nuance: naïve stretching-compression scenario would lead to destruction of the field (and does, in 2D)



AAS et al., *ApJ* **612**, 276 (2004)

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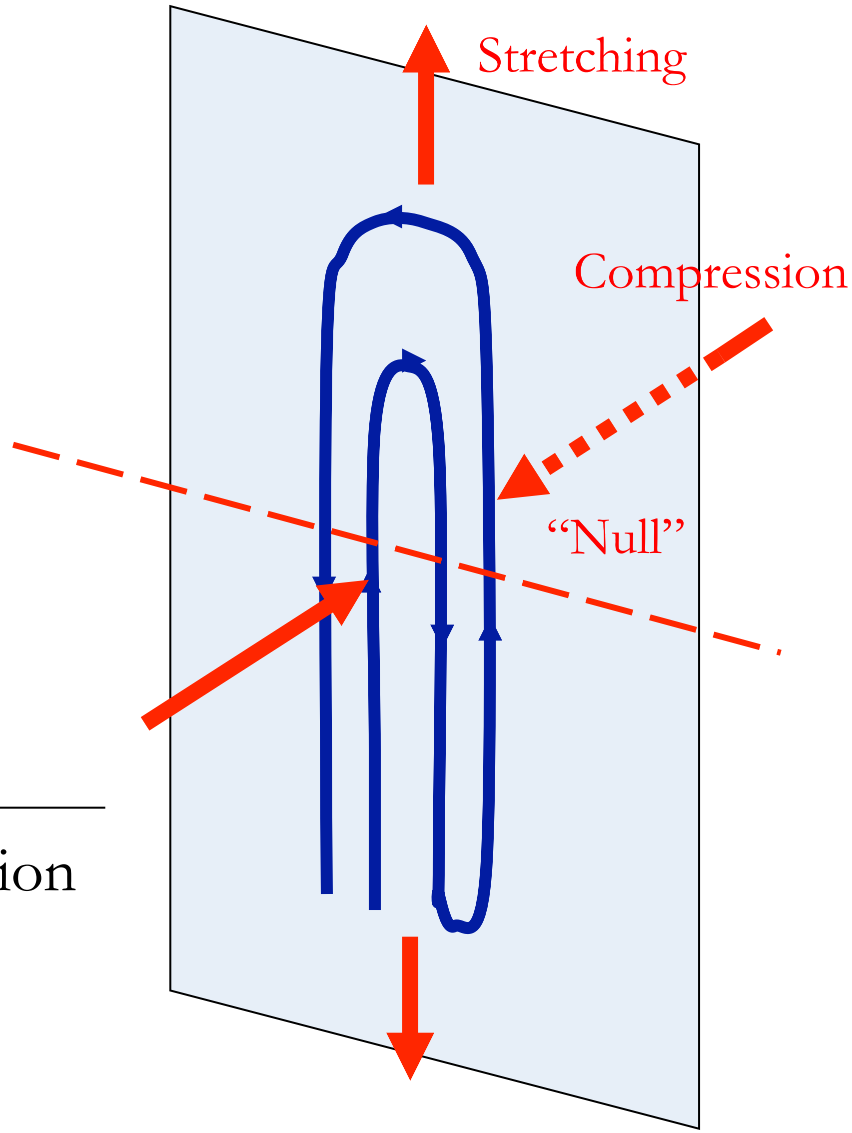

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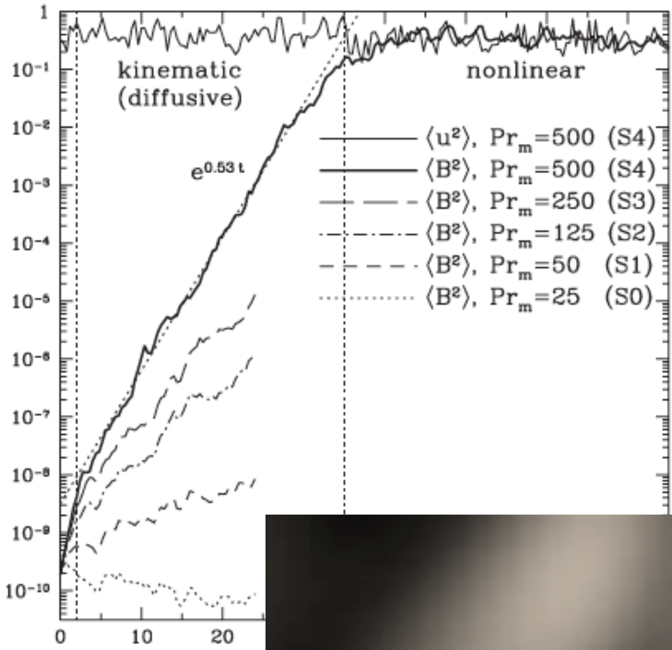
Nuance: naïve stretching-compression scenario would lead to destruction of the field (and does, in 2D)
In 3D, surviving folds are in the stretching-null plane [Zeldovich et al. 1984, *JFM* **144**, 1]

AAS et al., *ApJ* **612**, 276 (2004)

Numerical MHD Dynamo: SOLVED

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

All you need is a
chaotic flow
(and large enough Rm)



$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla \mathbf{u}) B \equiv \gamma B$$

The bottom line is that
turbulent dynamo works if
Rm > Rm_c ~ 50 to 200
(depending on Re)

AAS et al., *NJP* **9**, 300 (2007)

AAS et al., *ApJ* **612**, 276 (2004)

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First evidence was in 1981:

VOLUME 47, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1981

Helical and Nonhelical Turbulent Dynamos

M. Meneguzzi
Centre National de la Recherche Scientifique and Section d'Astrophysique, Division de la Physique, Centre d'Etudes Nucléaires de Saclay, F-91191 Gif-Sur-Yvette, France

and

U. Frisch
Centre National de la Recherche Scientifique, Observatoire de Nice, F-06007 Nice, France

and

A. Pouquet[a]
Centre National de la Recherche Scientifique, Observatoire de Meudon, F-92190 Meudon, France
(Received 13 April 1981)

Direct numerical simulations of three-dimensional magnetohydrodynamic turbulence with kinetic and magnetic Reynolds numbers up to 100 are presented. Spatially intermittent magnetic fields are observed in a flow with nonhelical driving. Small-scale helical driving produces strong large-scale nearly force-free magnetic fields.

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla \mathbf{u})B \equiv \gamma B$$

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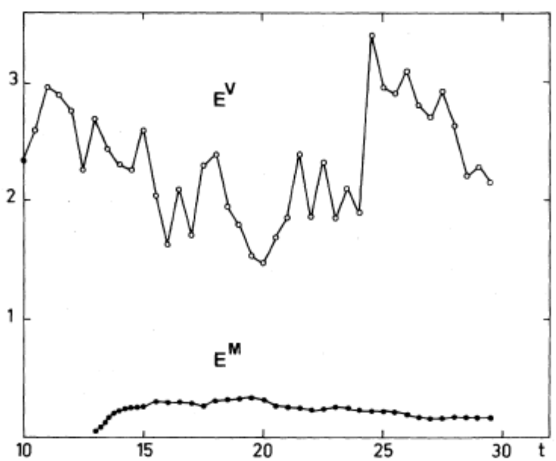
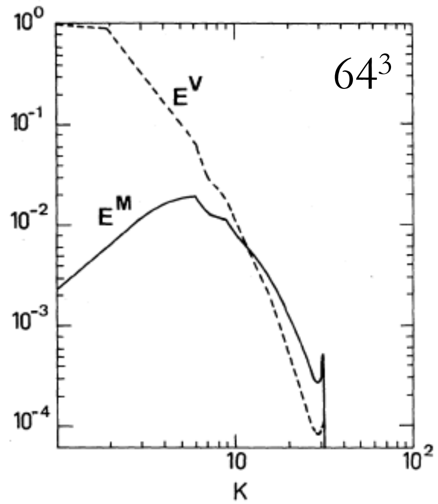




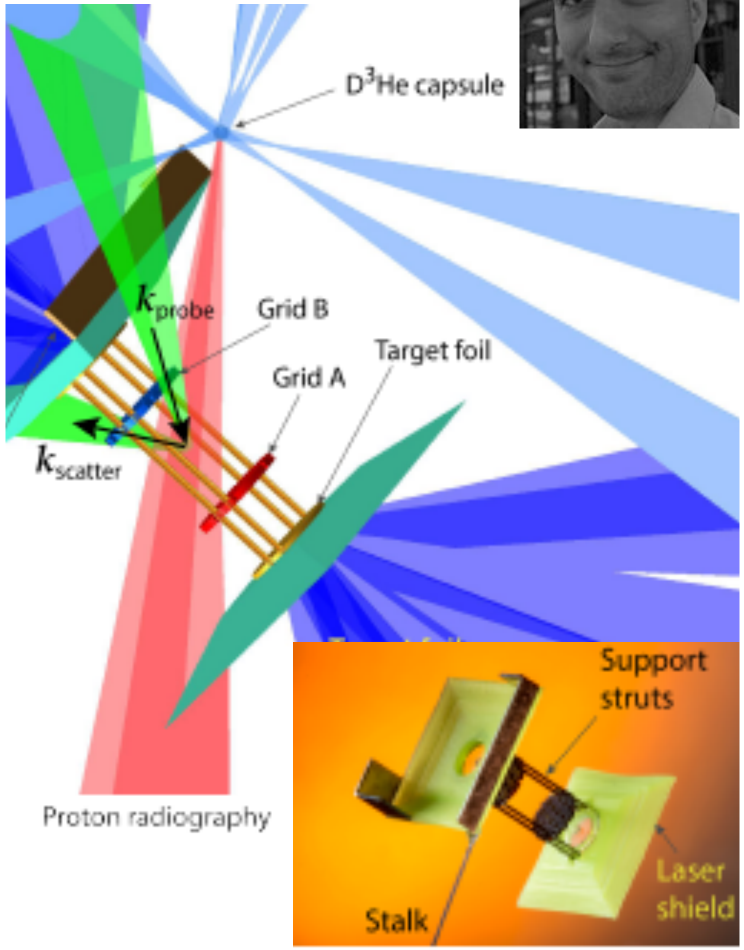
AAS et al., *NJP* **9**, 300 (2007)

Exp. MHD Dynamo: SOLVED? (G. Gregori)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

All you need is a
chaotic flow
(and large enough Rm)

Experiment on Omega laser:
two jet flows pass through grids,
collide, give turbulent cloud
with Re ~ 5000



The bottom line is that
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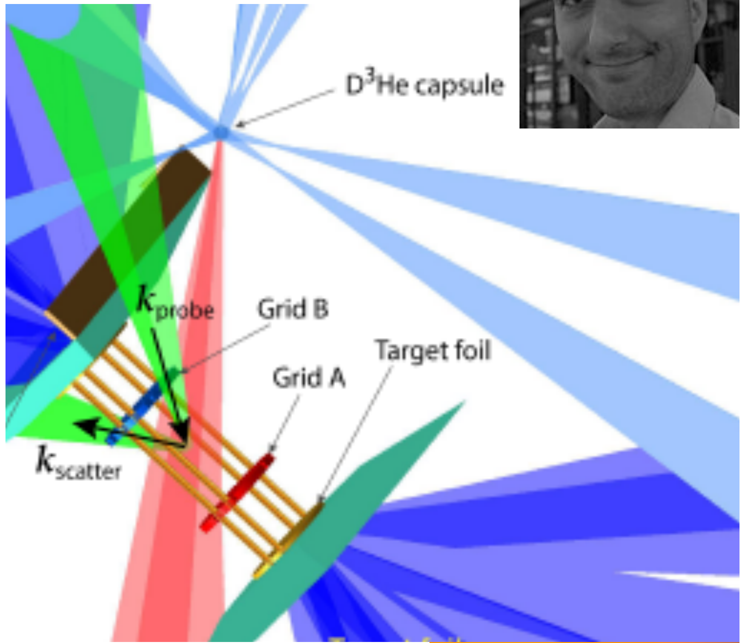
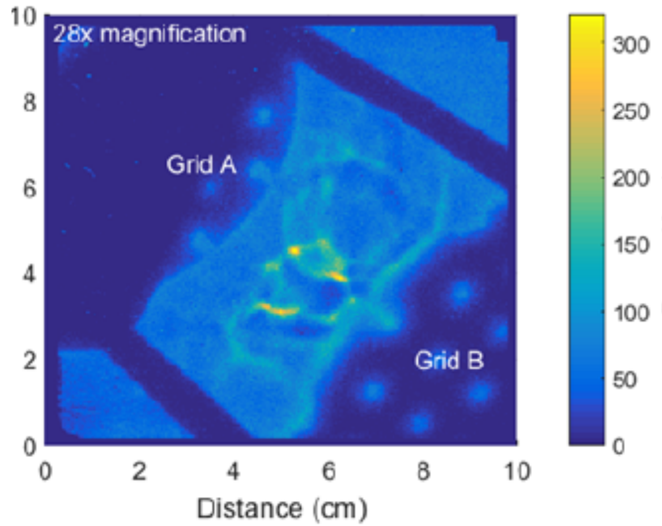


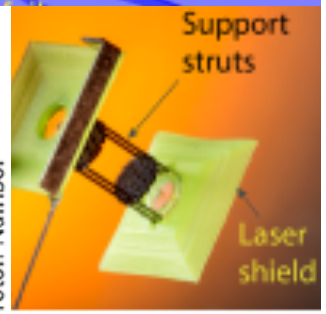
Experiment on Omega laser:

Rm~1000 > Rm$_c$

field amplified to B ~ 300kG

Details: come to HEDLA-2016!





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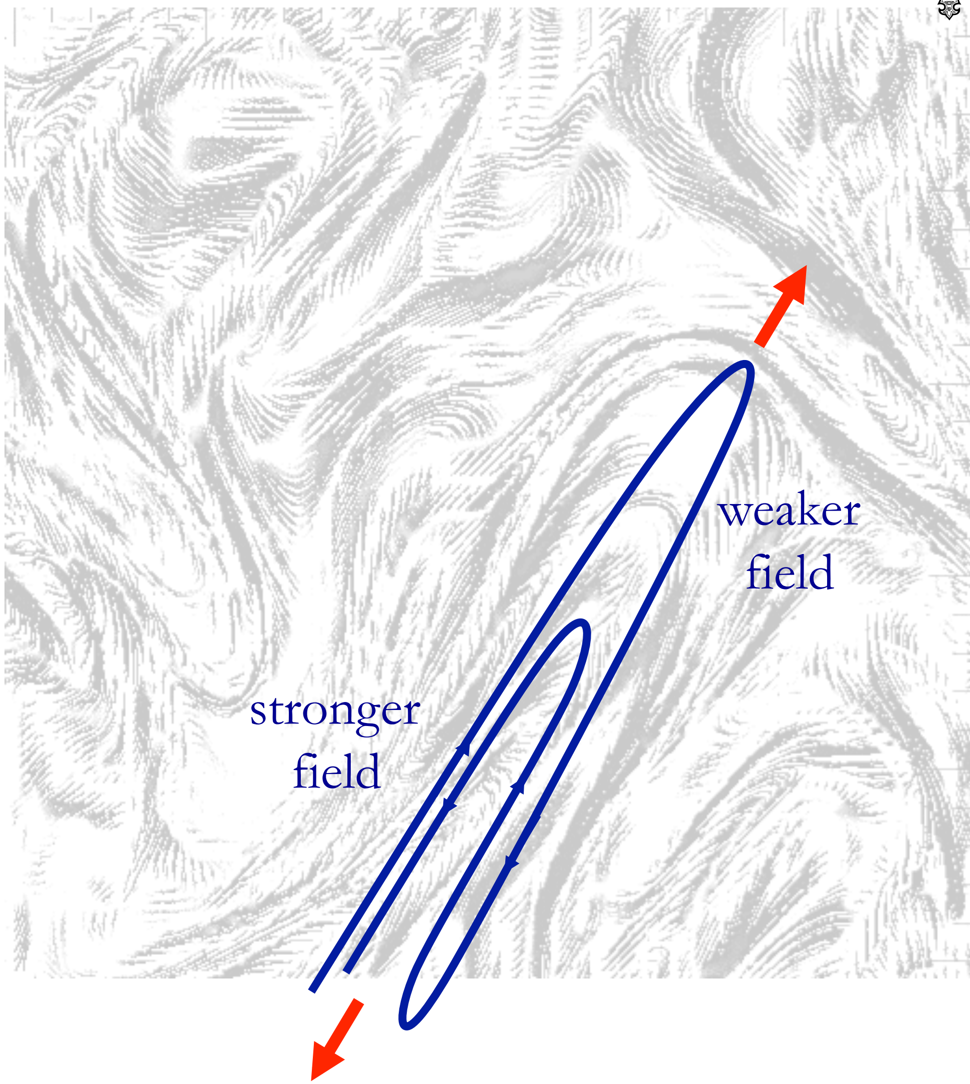
AAS et al., *NJP* **9**, 300 (2007)

Tzeferacos et al., in preparation (2016)
Meinecke et al., *PNAS* **112**, 8211 (2015)

In Fact, This Is All Irrelevant to Astro…

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla\mathbf{u})B \equiv \gamma B$$



Key effect: a succession of random stretchings (and un-stretchings)

AAS et al., *ApJ* **612**, 276 (2004)

Collisionless Plasma: Adiabatic Constraints

Changing magnetic field causes local pressure anisotropies:

$$\frac{1}{p_\perp}\frac{\mathrm{d}p_\perp}{\mathrm{d}t} = \frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t}$$

conservation of $\mu = v_\perp^2/B$

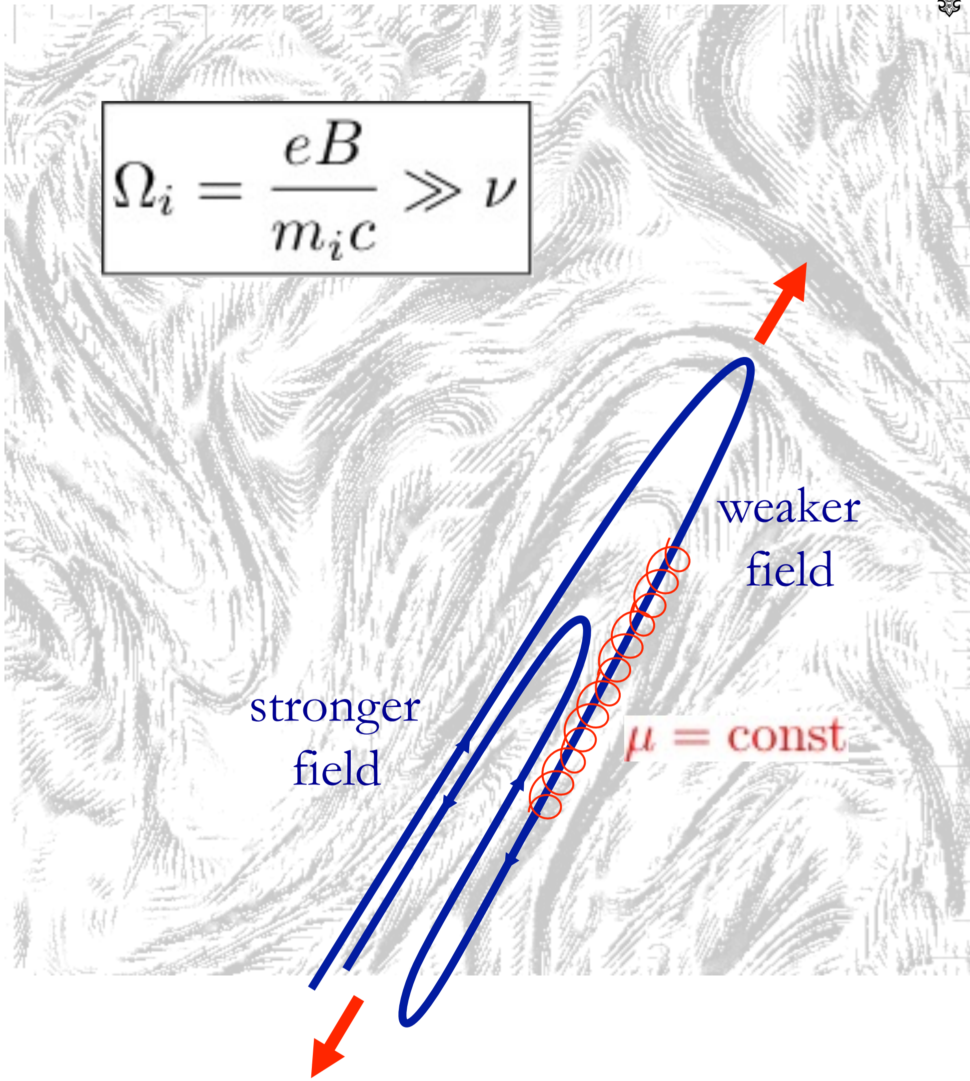
$$\frac{1}{2p_\parallel}\frac{\mathrm{d}p_\parallel}{\mathrm{d}t} = -\frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t}$$

conservation of $J = \oint \mathrm{d}\ell\, v_\parallel$

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla\mathbf{u})B \equiv \gamma B$$

It is very hard to change B in the face of these constraints!

[*CGL supports no dynamo action:* Santos-Lima et al. 2011, Proc. IAU No 274, 482]



Helander, Strumik & AAS, in preparation (2016)

Weak Collisions ➔ Pressure Anisotropy

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$$\frac{1}{p_\perp}\frac{\mathrm{d}p_\perp}{\mathrm{d}t} = \frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t} - \nu\frac{p_\perp - p_\parallel}{p_\perp}$$

conservation of $\mu = v_\perp^2/B$

$$\frac{1}{2p_\parallel}\frac{\mathrm{d}p_\parallel}{\mathrm{d}t} = -\frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t} - \nu\frac{p_\parallel - p_\perp}{p_\parallel}$$

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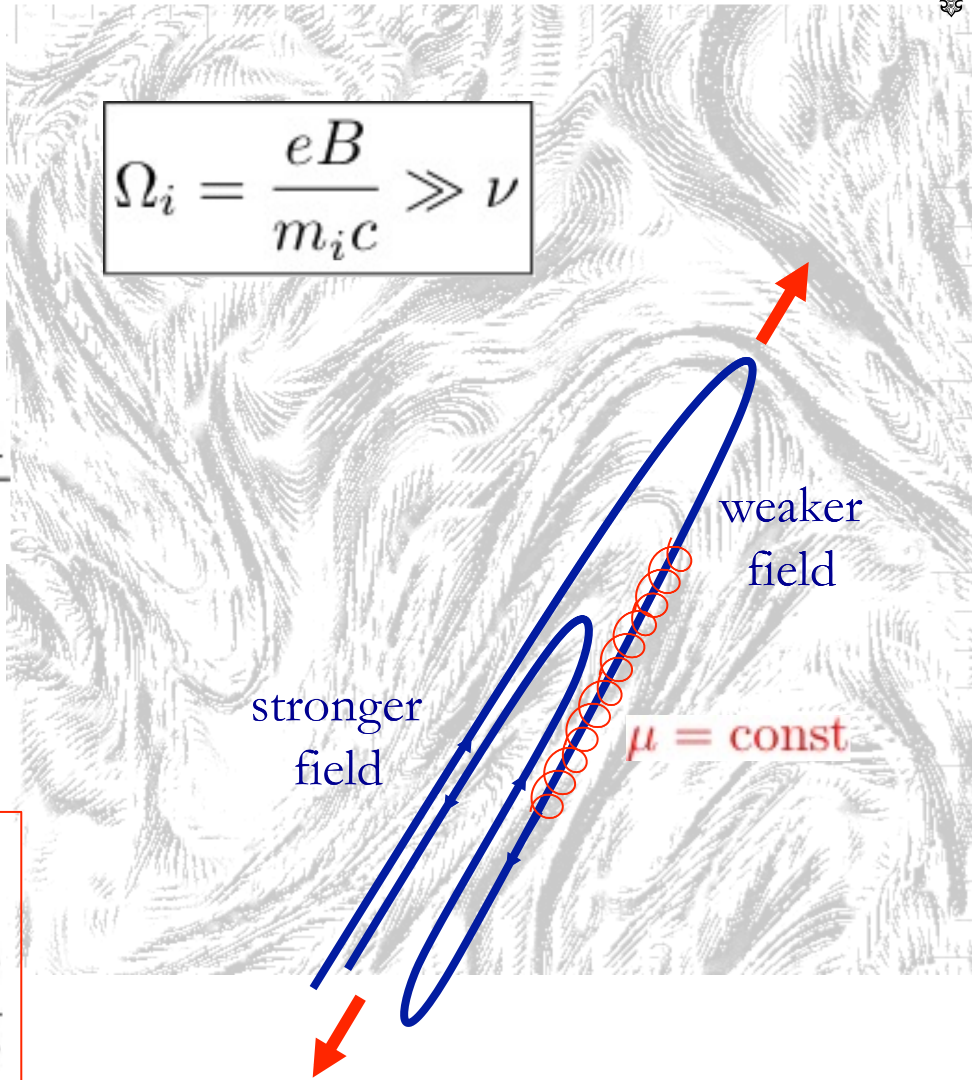
$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla\mathbf{u})B \equiv \gamma B$$

Typical pressure anisotropy:

$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu}$$

$$\Omega_i = \frac{eB}{m_i c} \gg \nu$$

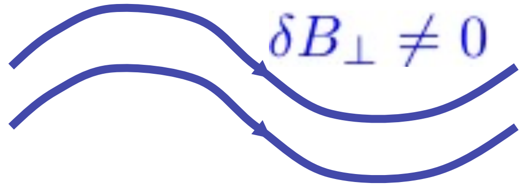


(compressions/rarefactions & heat fluxes are also sources of local pressure anisotropy)

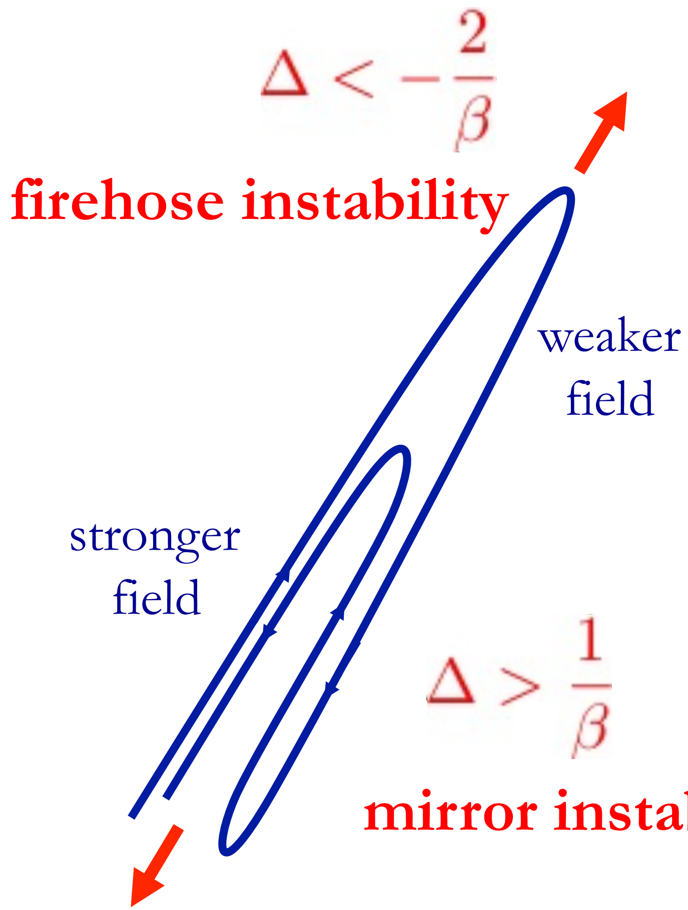
AAS et al., *ApJ* **629**, 139 (2005); *MNRAS* **405**, 291 (2010)

Pressure Anisotropy ➔ Microinstabilities

Instabilities are fast, small scale (~Larmor).
They are instantaneous compared
to "fluid" dynamics.



destabilised Alfvén wave



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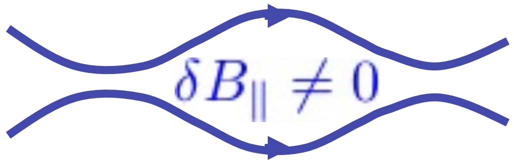

resonant instability

AAS et al., *ApJ* **629**, 139 (2005); *MNRAS* **405**, 291 (2010)

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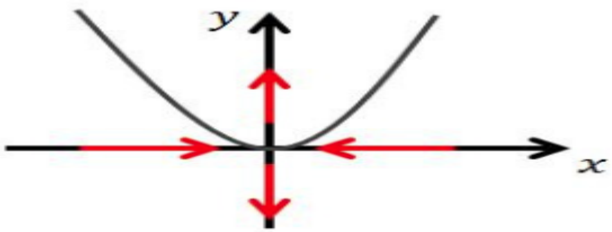

Firehose-unstable field lines
[Melville & AAS 2016]

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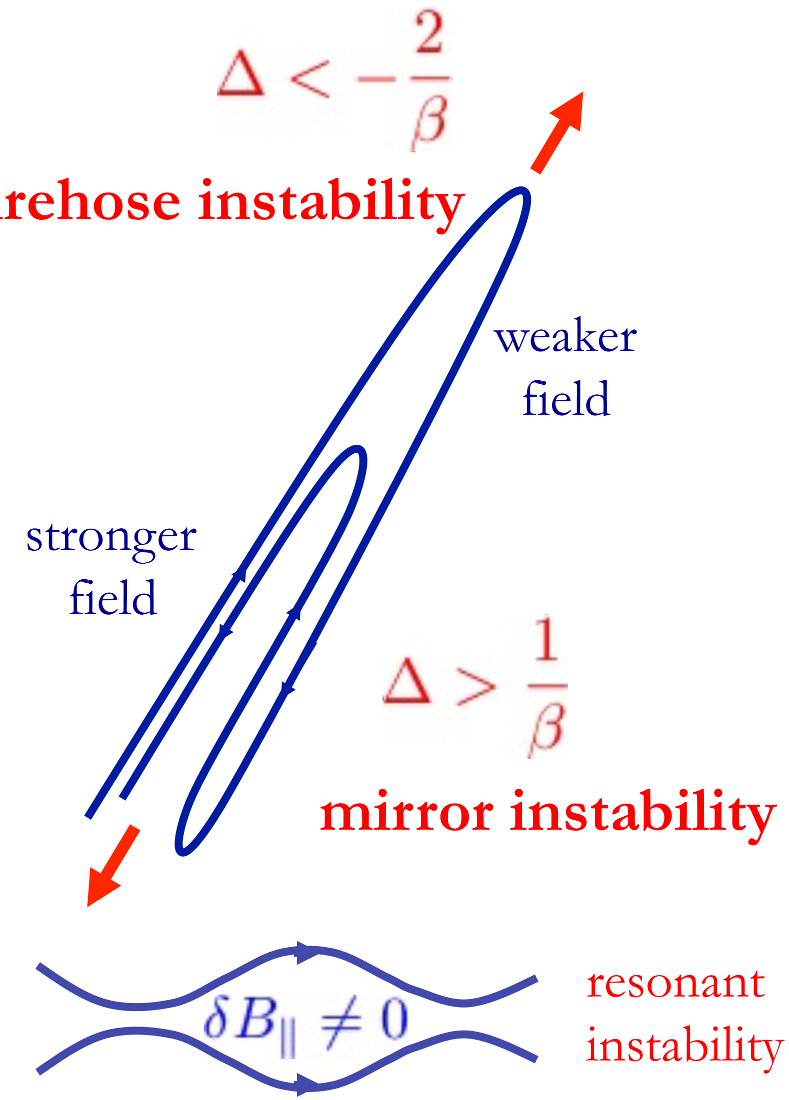

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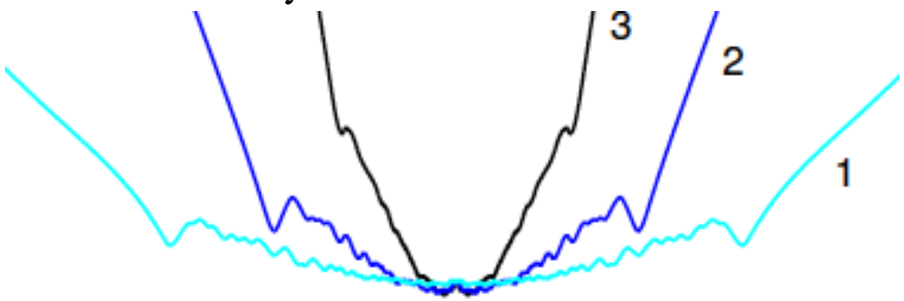

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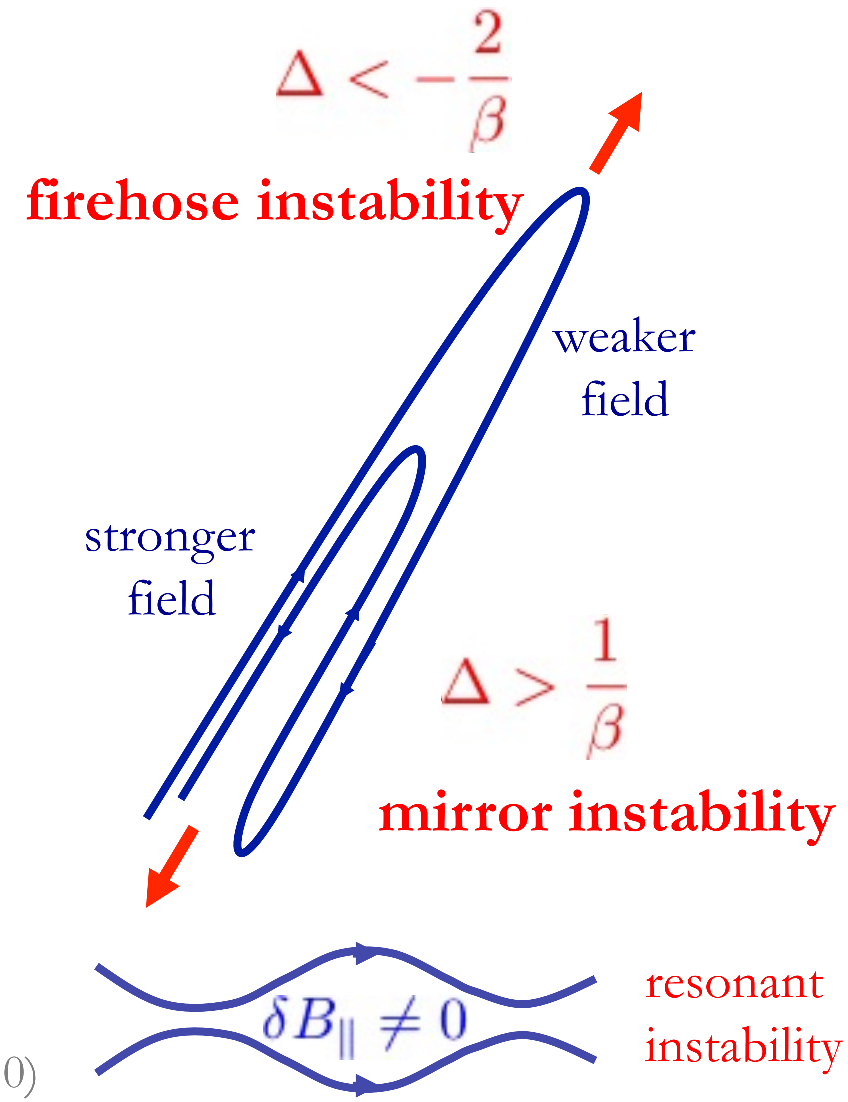
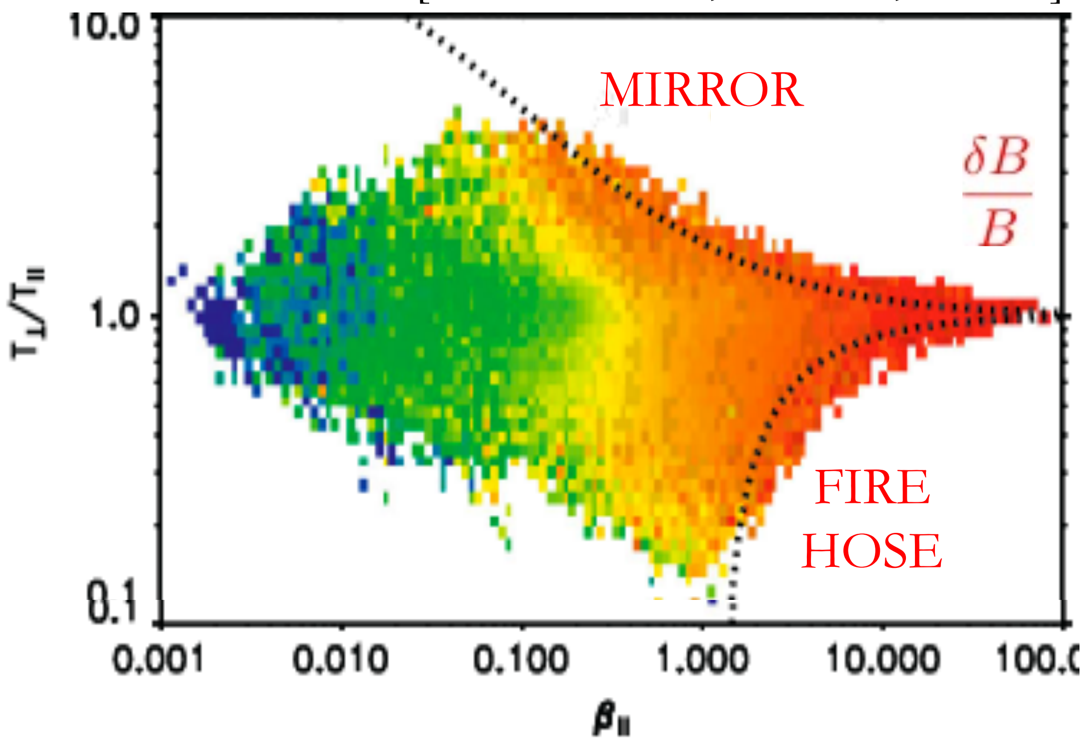


AAS et al., *ApJ* **629**, 139 (2005); *MNRAS* **405**, 291 (2010)

Marginal State At All Times?

In the **solar wind**:

[Bale et al. 2009, *PRL* **103**, 211101]



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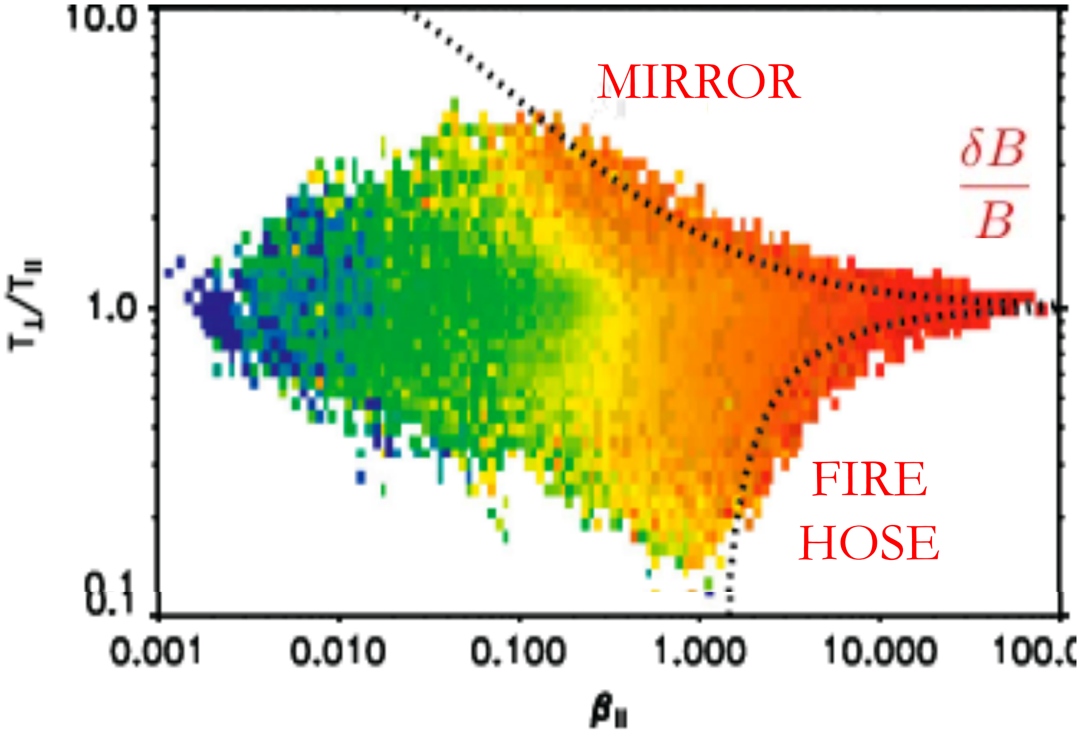
[Bale et al. 2009, *PRL* **103**, 211101]



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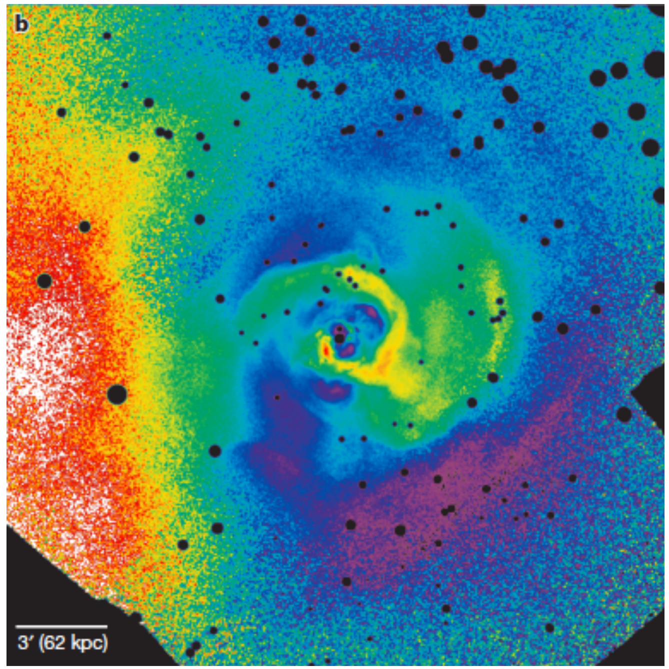
In **galaxy clusters**:

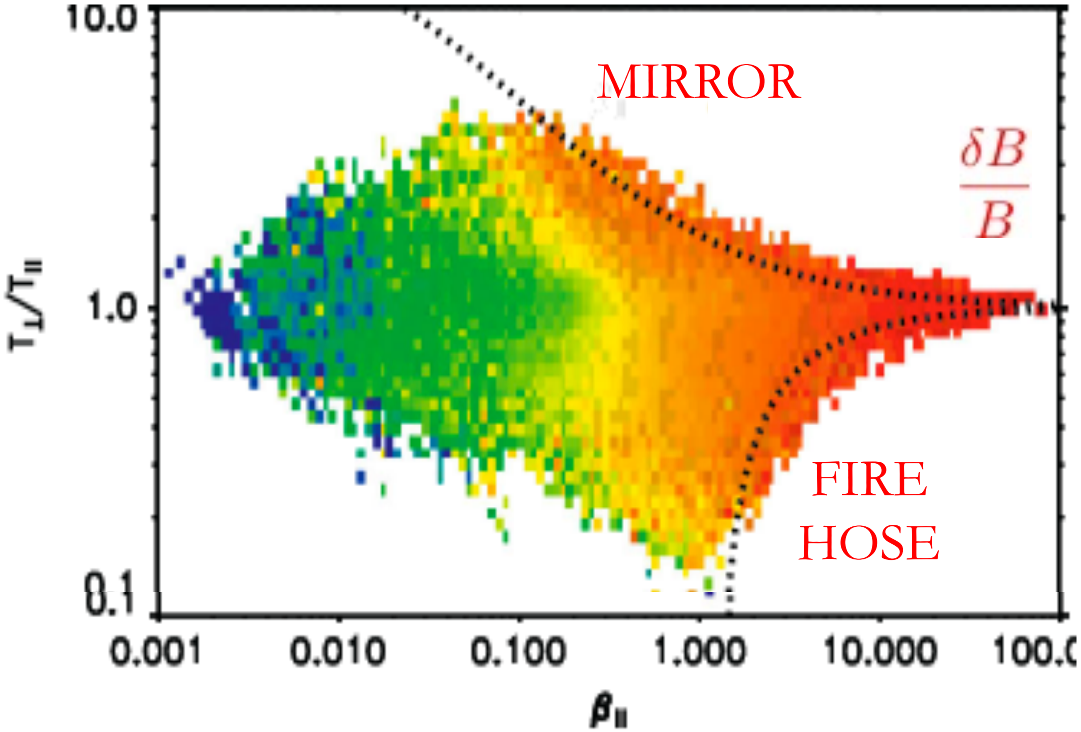
$$\beta \sim 100$$
$$\frac{\gamma}{\nu} \sim 0.01$$

[Image: Zhuravleva et al. 2014, *Nature* **515**, 85]



Marginal State At All Times?

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?



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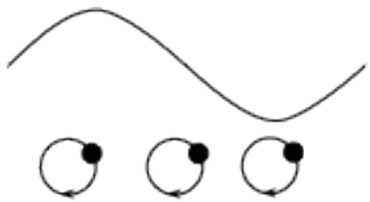
Mogavero & AAS,
MNRAS **440**, 3226 (2014)

Effective Closure Options

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

Way to keep const rms B needed for this

Option I: Suppress stretching



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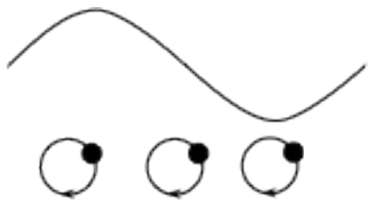

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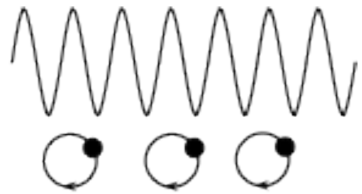
Option II: Enhance collisionality



Anomalous scattering of particles by Larmor scale fluctuations needed for this

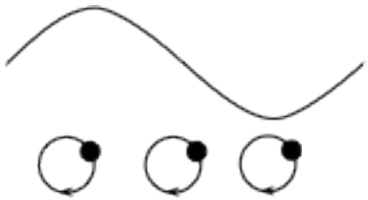
Mogavero & AAS, *MNRAS* **440**, 3226 (2014)

PLASMA DYNAMO: UNSOLVED?

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

In view of these complications, **does dynamo work in a weakly collisional plasma?**

Way to keep const rms B needed for this



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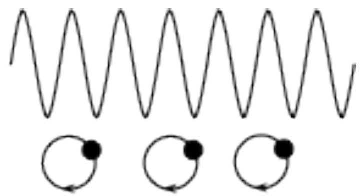
$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

Option II: Enhance collisionality



Anomalous scattering of particles by Larmor scale fluctuations needed for this

Mogavero & AAS, *MNRAS* **440**, 3226 (2014)

Num. Plasma Dynamo: SOLVED (F. Rincon)

Hybrid kinetic system solved by a Vlasov code (grid):

$$\frac{\partial f_i}{\partial t} + \mathbf{v}\cdot\nabla f_i + \left[\frac{e}{m_i}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) + \frac{\mathbf{F}}{m_i}\right]\cdot\frac{\partial f_i}{\partial \mathbf{v}} = 0$$

$$\mathbf{E} = -\frac{T_e\nabla n_e}{e n_e} - \frac{\mathbf{u}_e\times\mathbf{B}}{c} + \frac{4\pi\eta}{c^2}\mathbf{j}$$

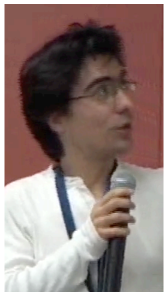
isothermal, massless electron fluid

Valentini et al.
JCP **225**, 753 (2007)

$$\mathbf{u}_e = \mathbf{u}_i - \frac{\mathbf{j}}{e n_e}$$

$$\mathbf{j} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

Force randomly



$$n_e = n_i$$

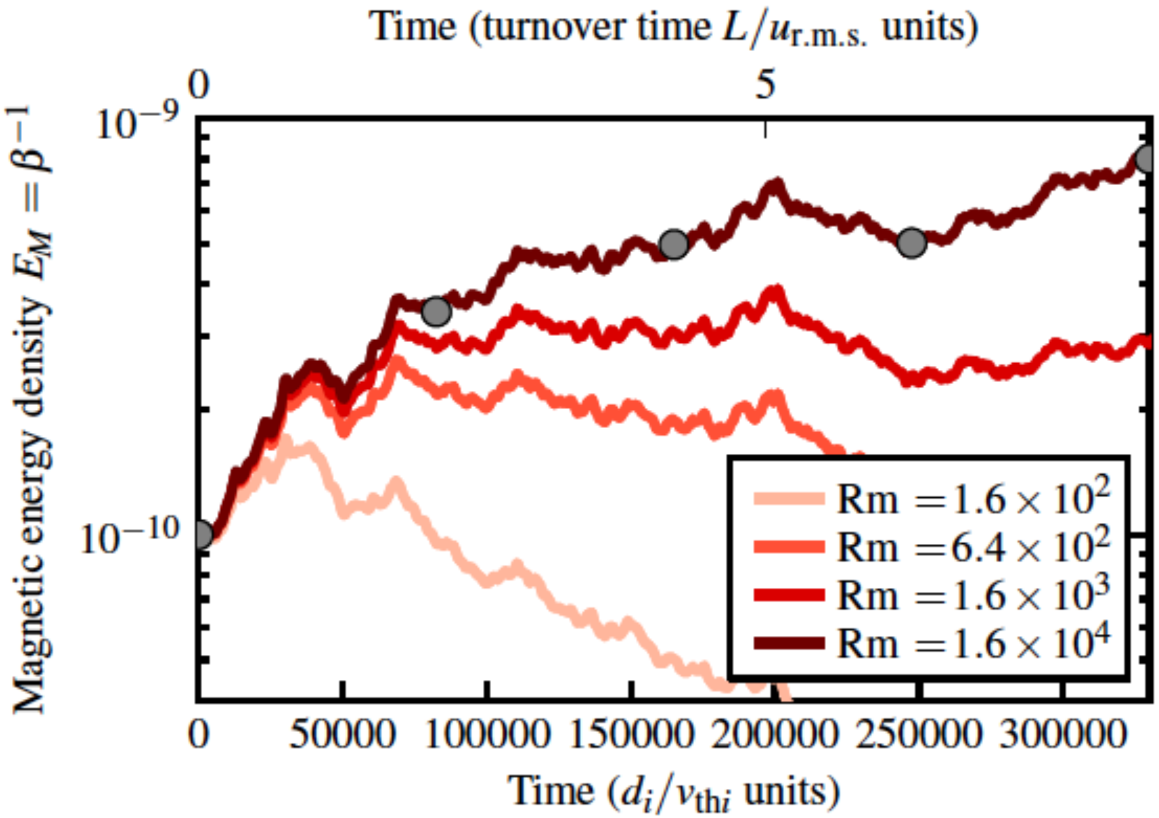
Increase $\mathrm{Rm} = \frac{u_{\mathrm{rms}}L}{\eta}$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}$$

Will magnetic energy grow?

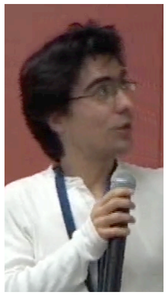
Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Num. Plasma Dynamo: SOLVED (F. Rincon)



UNMAGNETISED

$$\beta = 10^{10} \quad \frac{\rho_i}{L} \simeq 16$$



Dynamo works!

$\mathrm{Rm_c} \sim 10^3$

3D3V $64^3 \times 51^3$

$L = 2000\pi d_i$

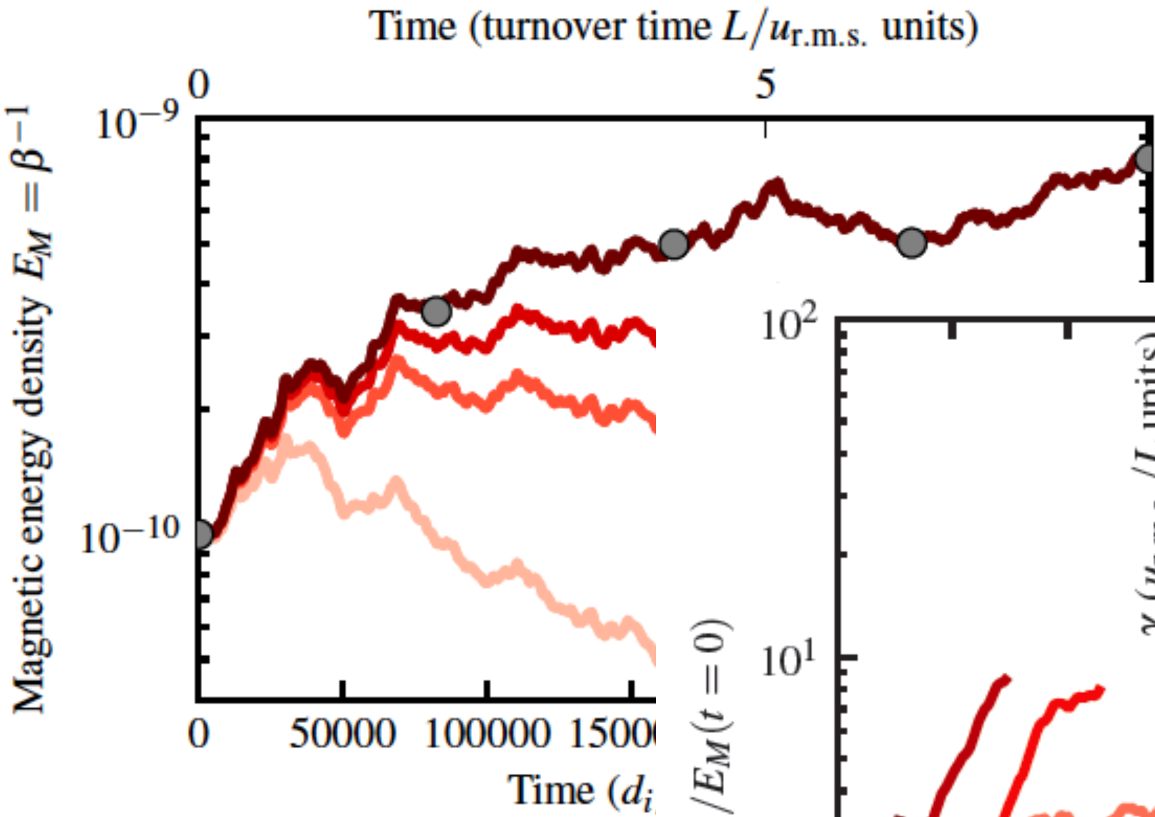
$v_{\mathrm{max}} = 5 v_{\mathrm{th}i}$

$u_{\mathrm{rms}} \sim 0.2 v_{\mathrm{th}i}$

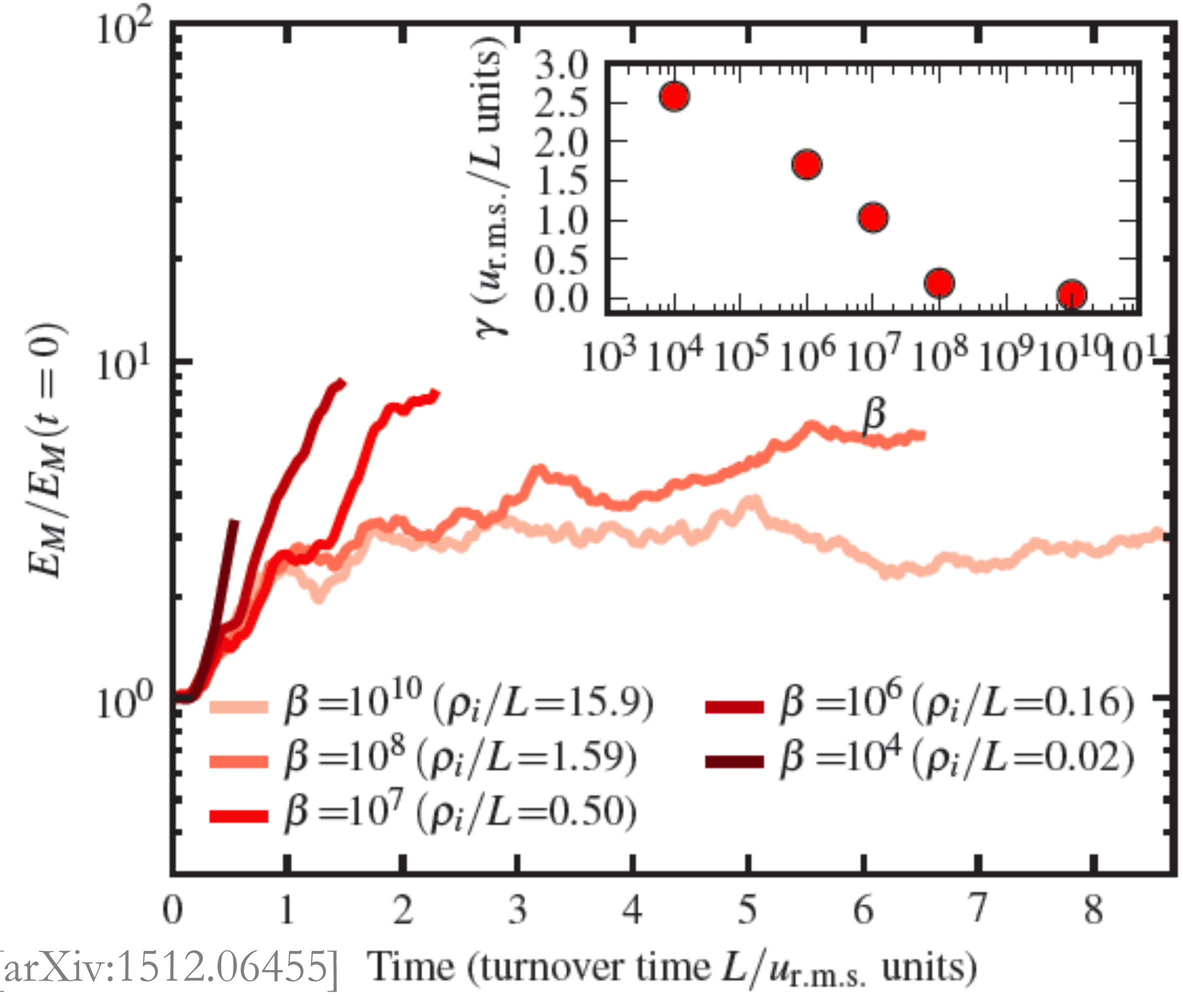
$10^6 - 10^7$ CPU hrs

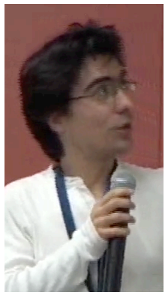
Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Num. Plasma Dynamo: SOLVED (F. Rincon)



Dynamo self-accelerates!



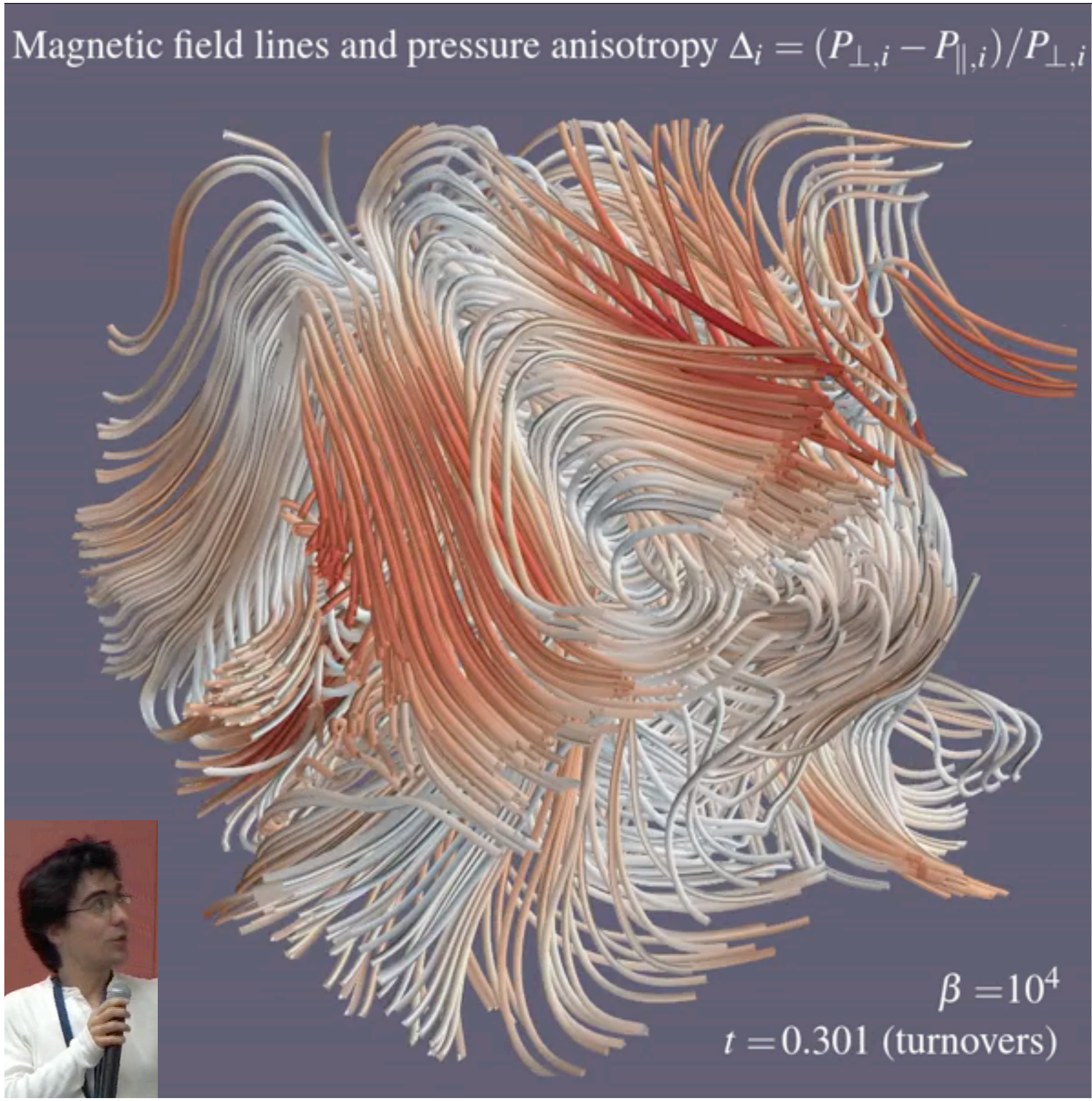


Dynamo works!

$\mathrm{Rm_c} \sim 10^3$

Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Num. Plasma Dynamo: UNSOLVED (F. Rincon)



FULLY MAGNETISED

$$\beta = 10^4 \quad \frac{\rho_i}{L} \simeq 0.02$$

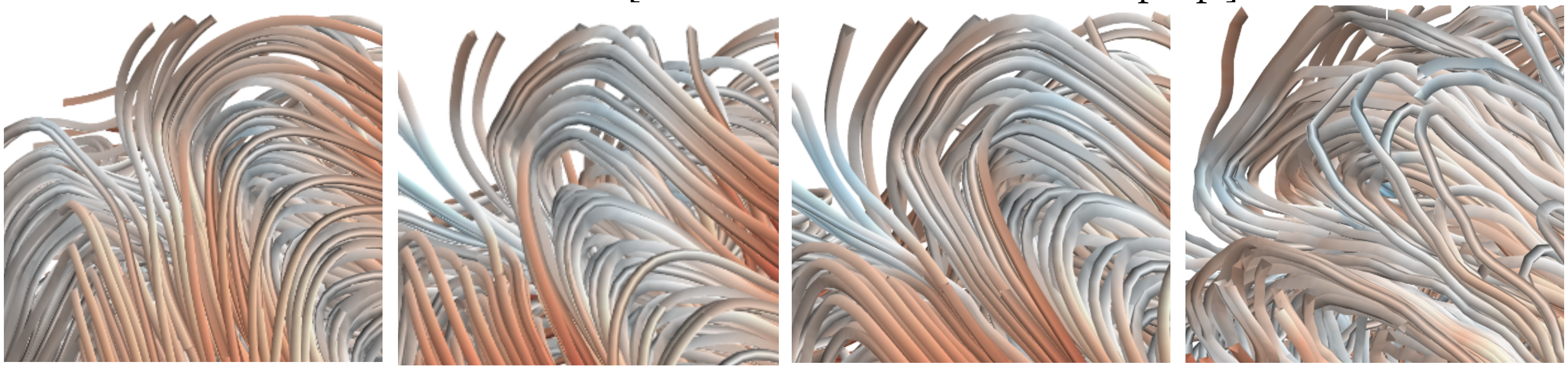
This couldn't be run for a very long time, so, in fact, it still belongs to the **(semi-)UNSOLVED** *category*
(but **will be solved** imminently by M. Kunz & D. St-Onge)

Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Num. Plasma Dynamo: UNSOLVED (F. Rincon)

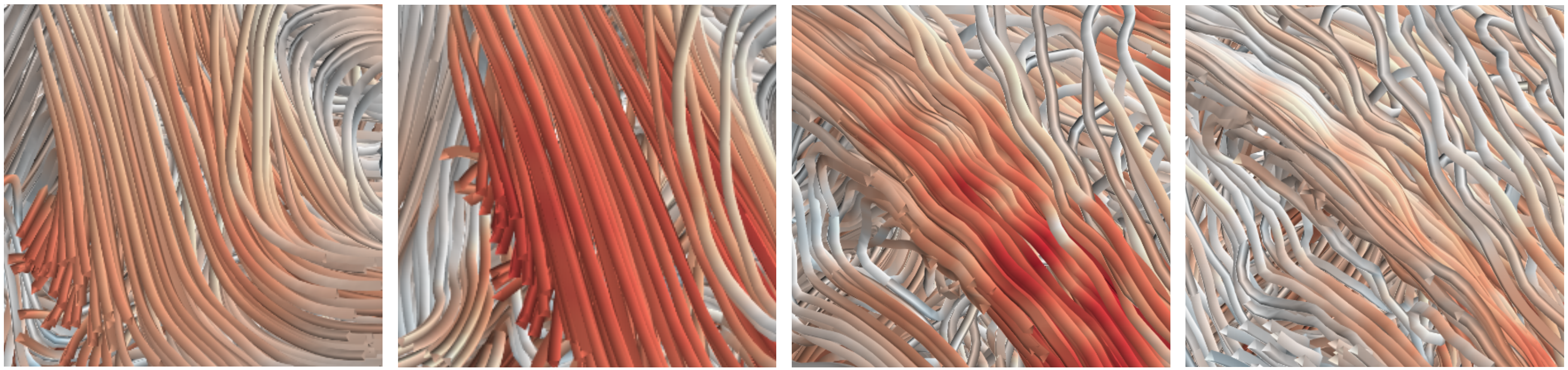
Firehoses in bends: *note square bends*

[cf. Melville & AAS 2016, in prep.]



Mirrors in stretched folds: *"bubbles" filled with trapped particles*

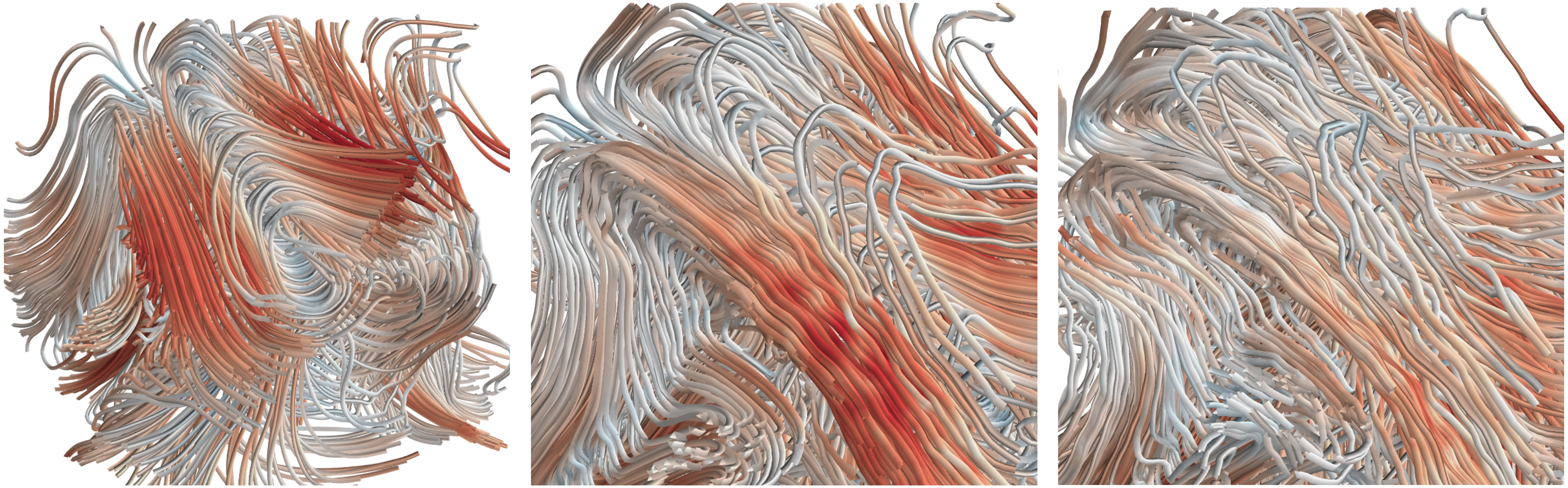
[cf. Rincon, AAS & Cowley 2015, MNRAS 447, L45]



Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Num. Plasma Dynamo: UNSOLVED (F. Rincon)

Pressure anisotropy relaxes in some self-consistent way:

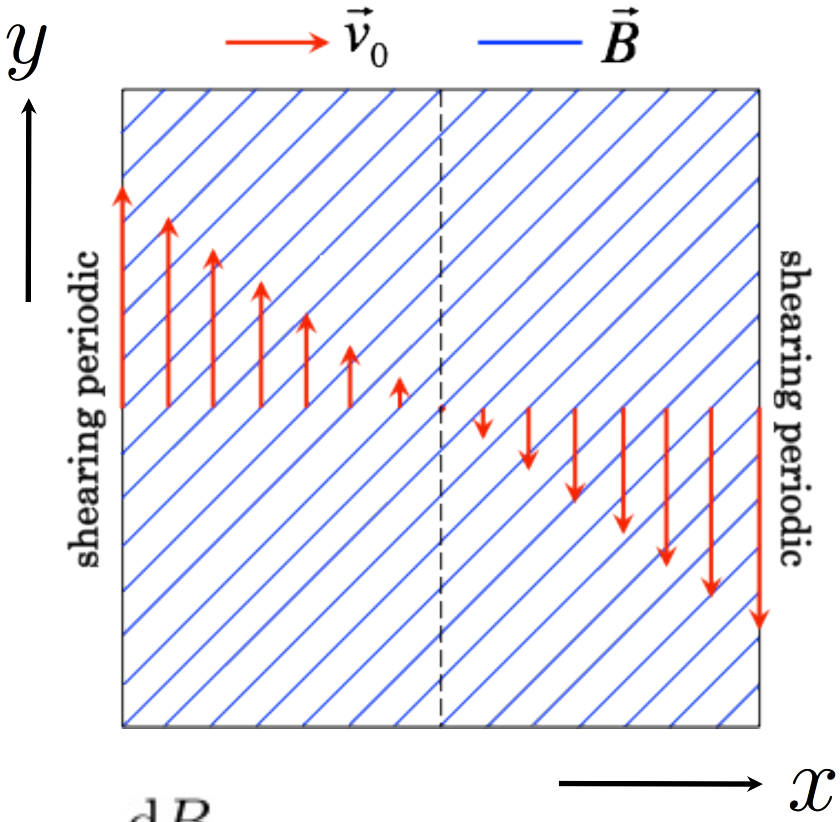


It would be fascinating to follow this dynamo for a long time and see how the macro-micro scale interaction works, how anisotropy adjusts, etc. But this is currently unaffordable in 3D3V.

Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Instabilities in a Shearing Box (M. Kunz)

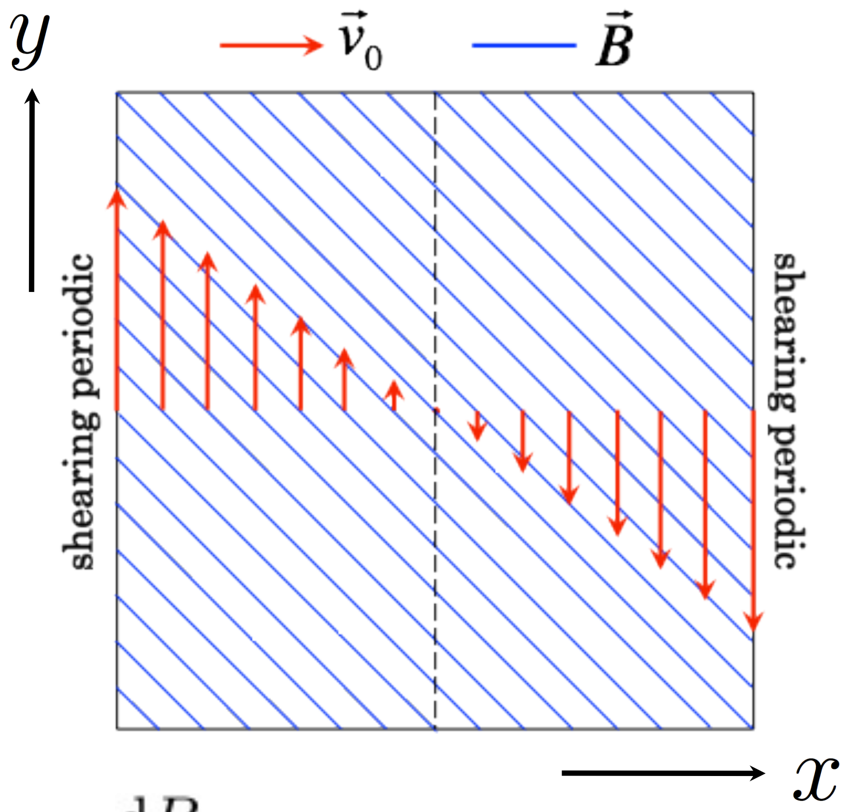
FIREHOSE



$$\frac{\mathrm{d}B}{\mathrm{d}t} < 0$$

$$\Rightarrow \quad \Delta = \frac{p_\perp - p_\|}{p} < 0$$

MIRROR



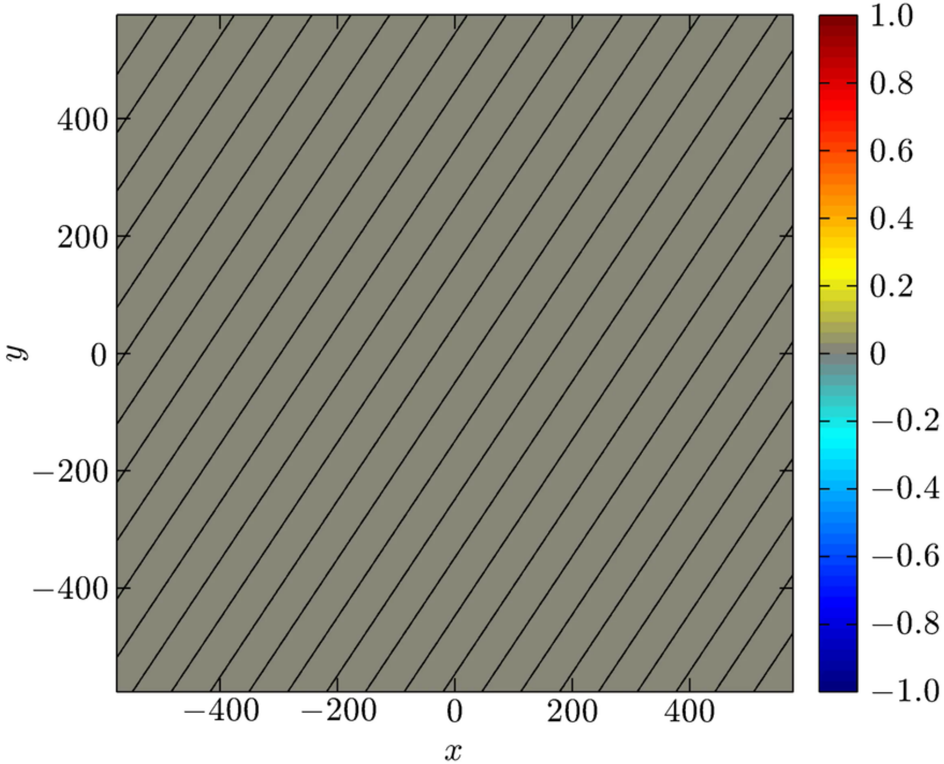
$$\frac{\mathrm{d}B}{\mathrm{d}t} > 0$$

$$\Rightarrow \quad \Delta = \frac{p_\perp - p_\|}{p} > 0$$

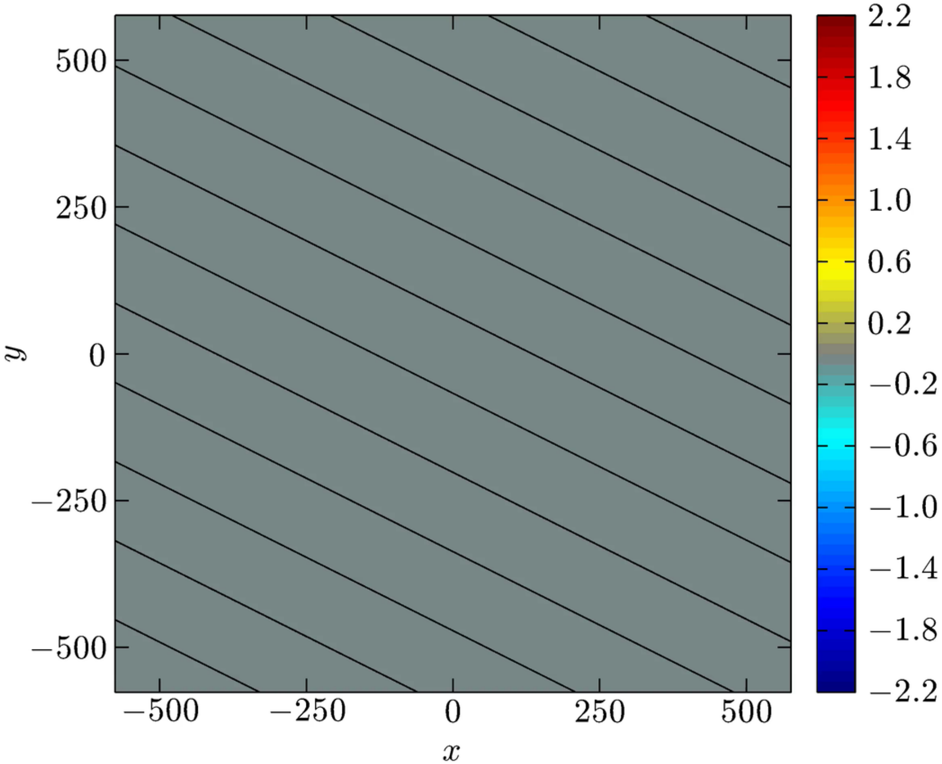
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Instabilities in a Shearing Box (M. Kunz)

FIREHOSE



MIRROR



$$\frac{\mathrm{d}B}{\mathrm{d}t} < 0$$

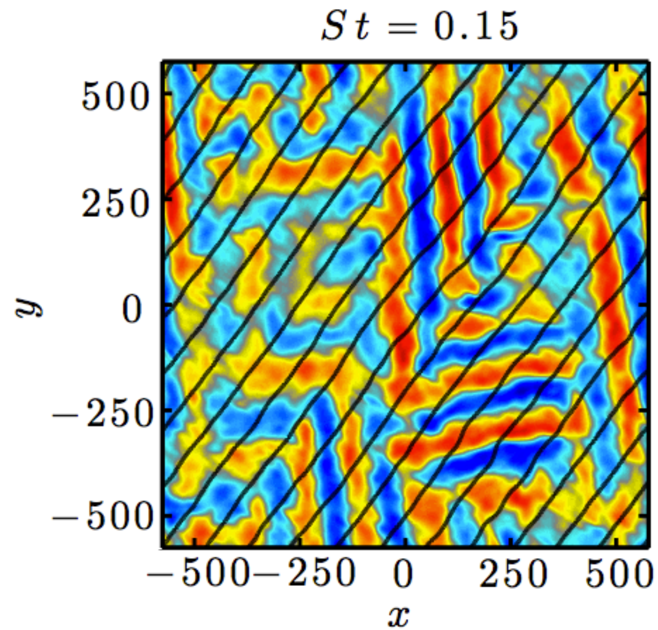
$$\Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} < 0$$

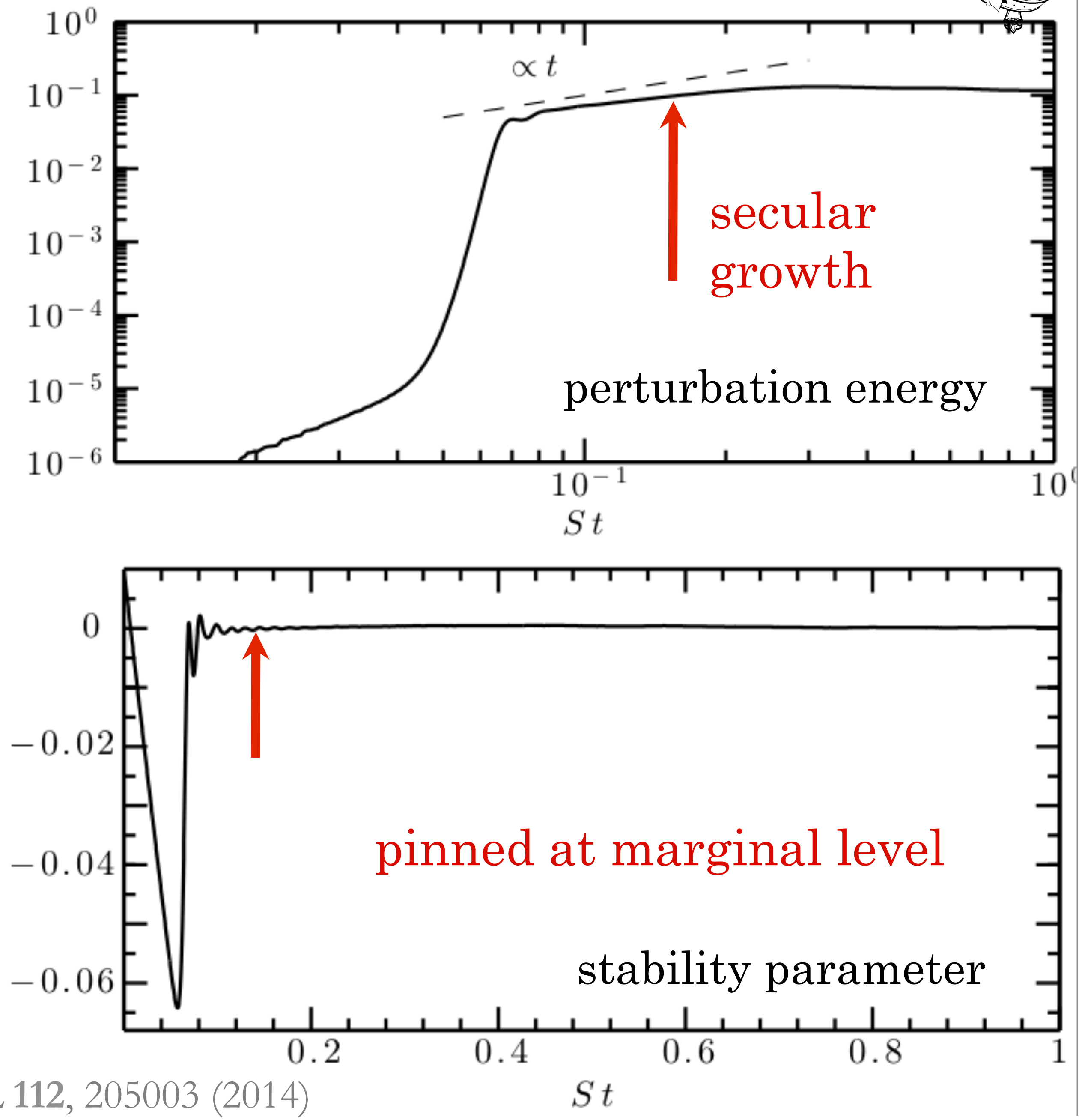
$$\frac{\mathrm{d}B}{\mathrm{d}t} > 0$$

$$\Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} > 0$$

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Firehose Instability: Secular





Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Firehose Instability: Secular

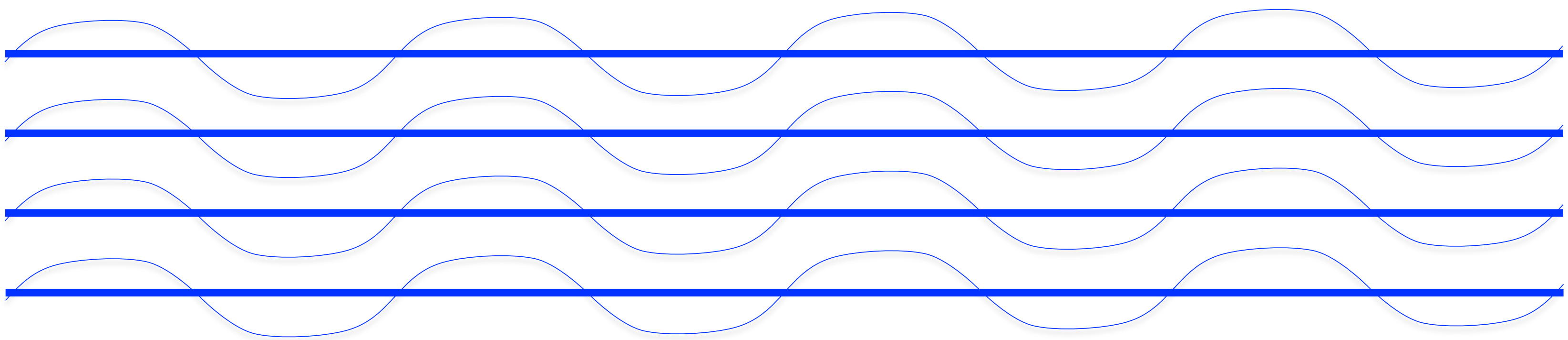
$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \frac{\overline{\mathrm{d}\ln B}}{\mathrm{d}t} = \int^t \mathrm{d}t' \left(-3\left|\frac{\mathrm{d}\ln B_0}{\mathrm{d}t}\right| + \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability



AAS et al., *PRL* **100**, 081301 (2008)

Rosin et al., *MNRAS* **413**, 7 (2011)

Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

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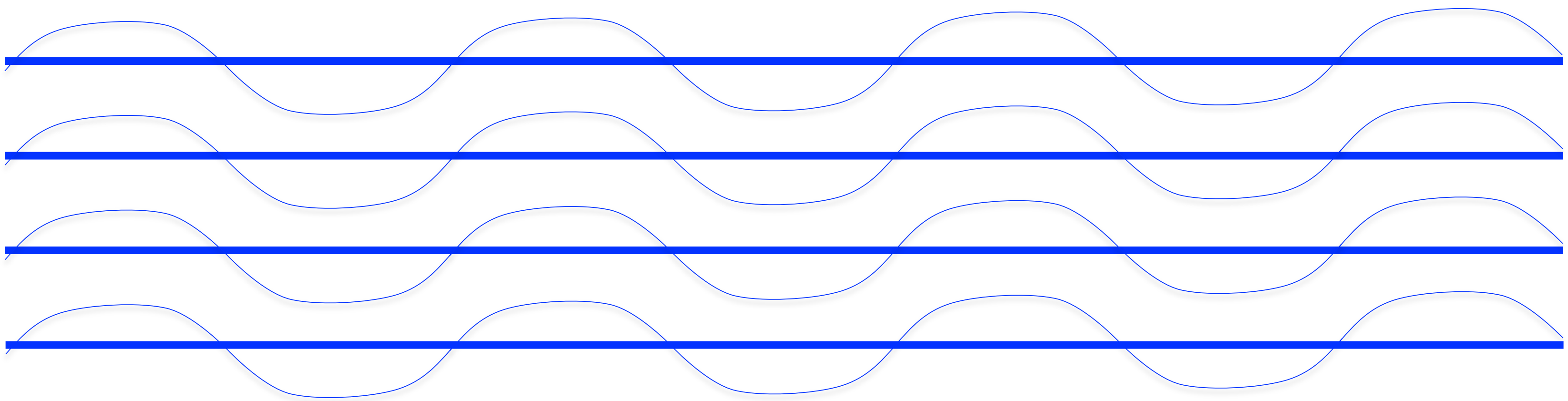
pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability



AAS et al., *PRL* **100**, 081301 (2008)
Rosin et al., *MNRAS* **413**, 7 (2011)
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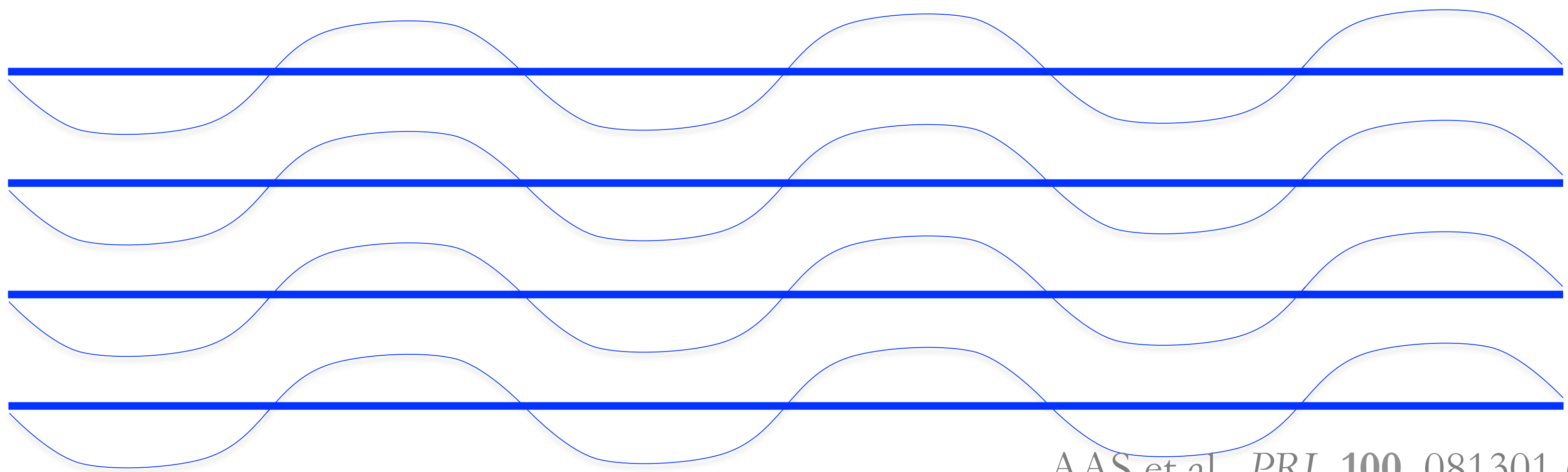
pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability



AAS et al., *PRL* **100**, 081301 (2008)

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Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

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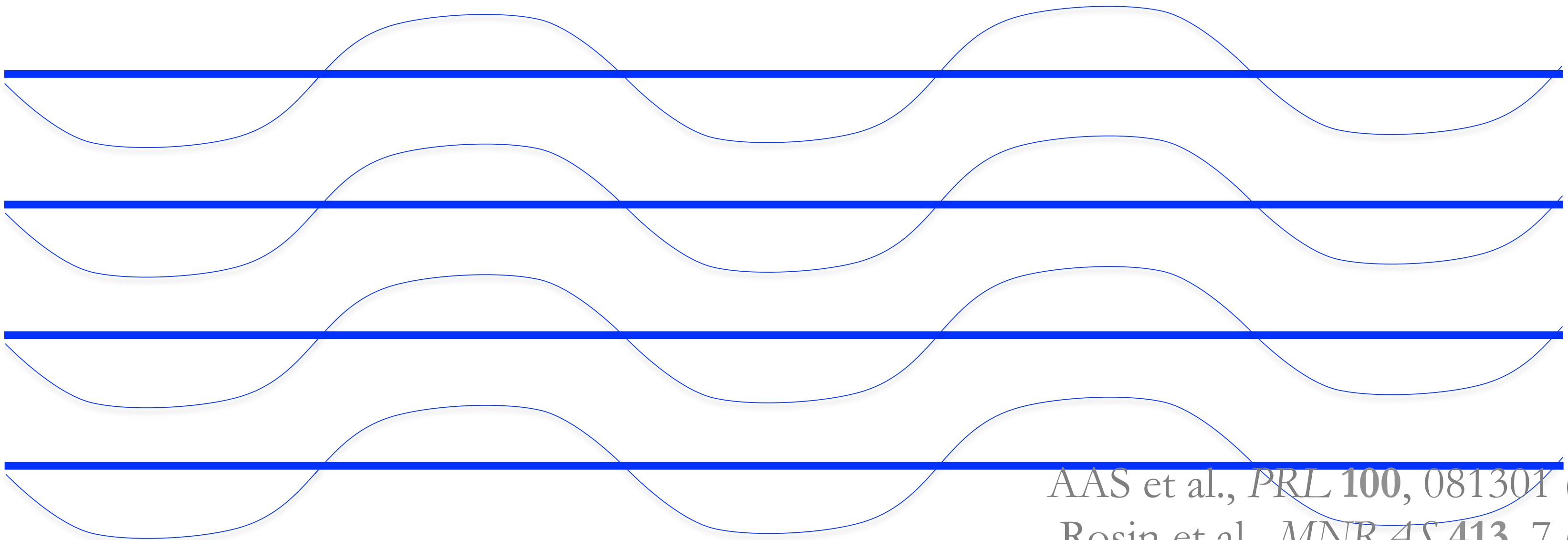
$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = \int^t \mathrm{d}t' \left(\underbrace{-3\left|\frac{\mathrm{d}\ln B_0}{\mathrm{d}t}\right|}_{\text{pressure anisotropy driven by shear}} + \underbrace{\frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2}}_{\text{pressure anisotropy from firehose}} \right) \to \underbrace{-\frac{2}{\beta}}_{\text{marginal stability}}$$



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Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

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$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t dt' \, \overline{\frac{d\ln B}{dt}} = \int^t dt' \left(-3\left|\frac{d\ln B_0}{dt}\right| + \frac{3}{2}\frac{d}{dt}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta}$$

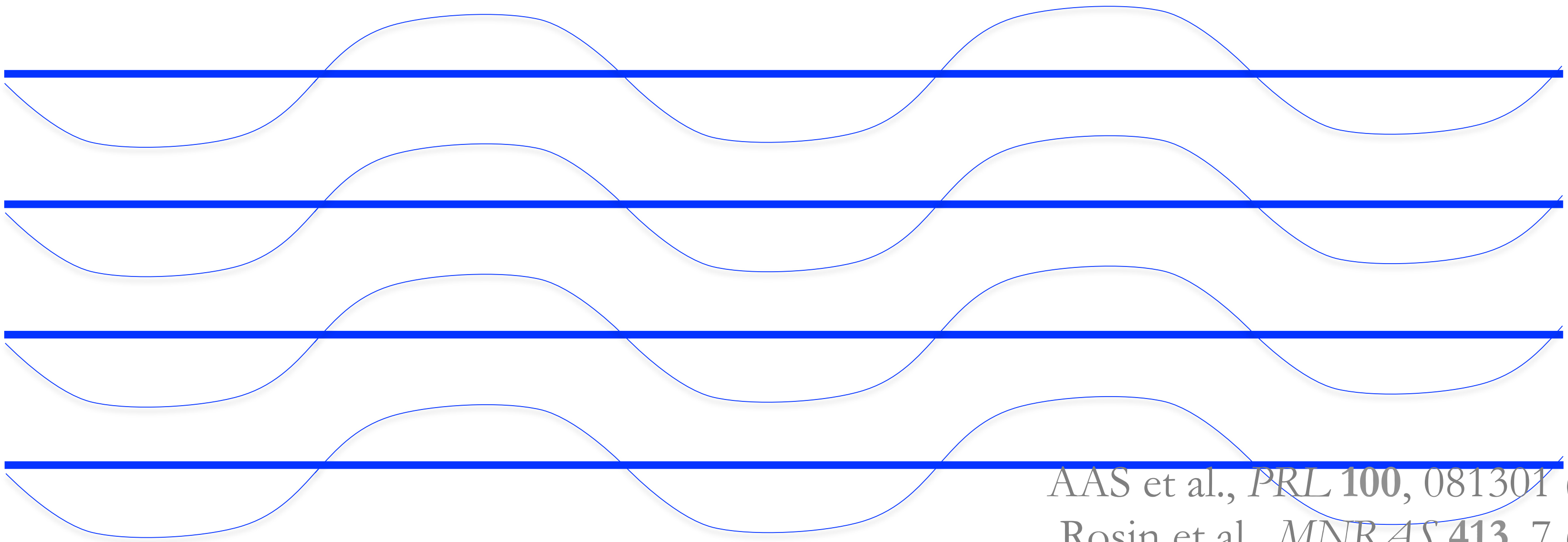
pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability

$$\frac{3}{2}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} = 3S\int^t dt' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{2}{\beta} \sim St$$

secular growth

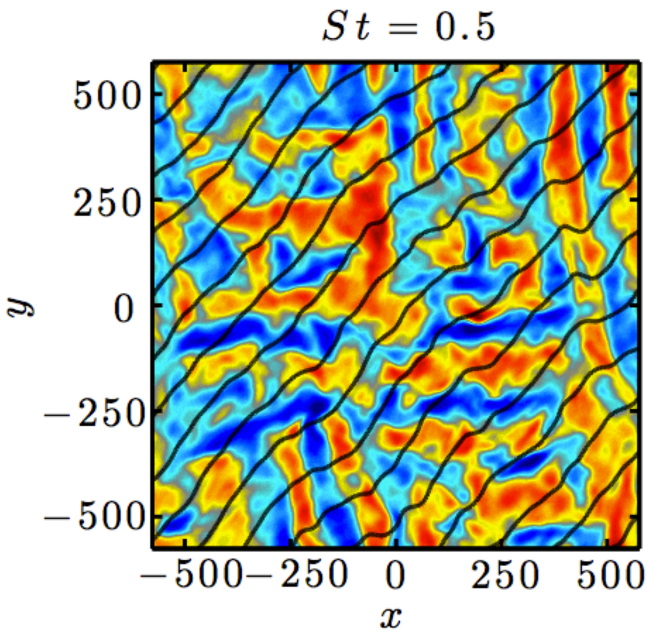


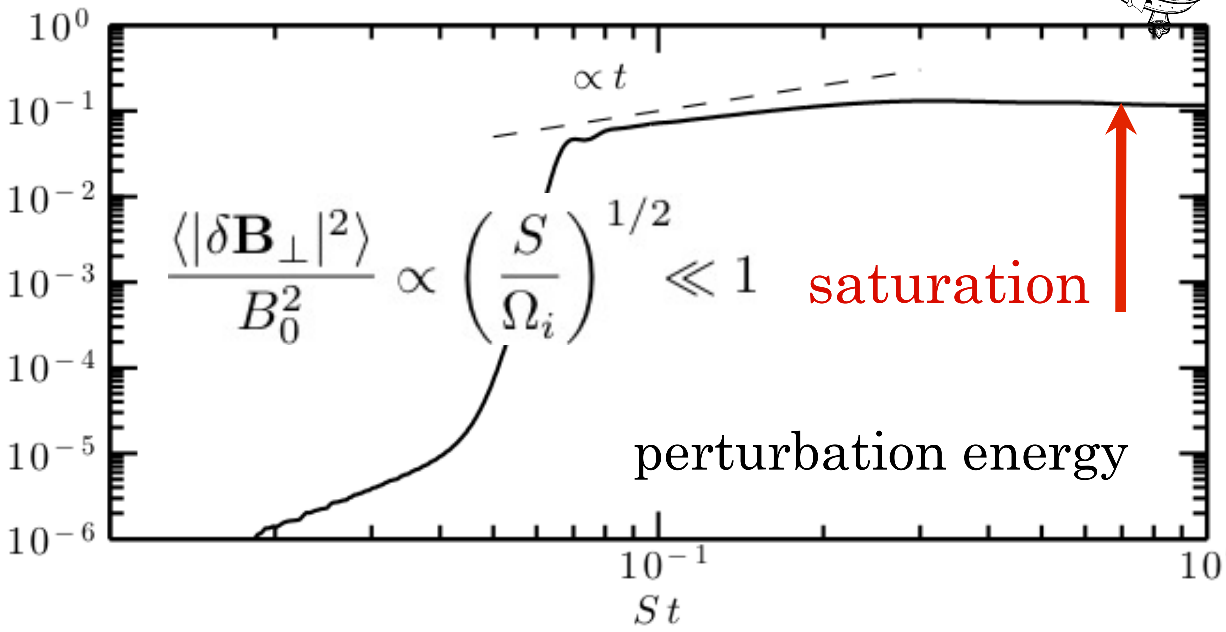
AAS et al., *PRL* **100**, 081301 (2008)

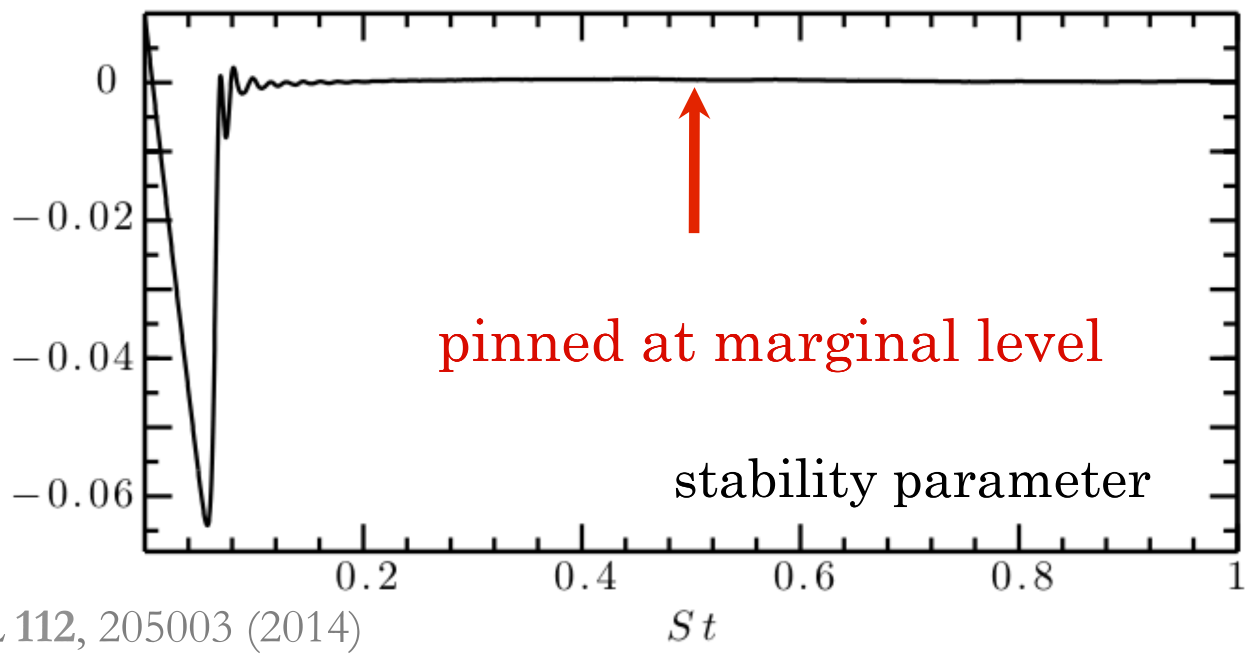
Rosin et al., *MNRAS* **413**, 7 (2011)

Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

Firehose Instability: Saturated

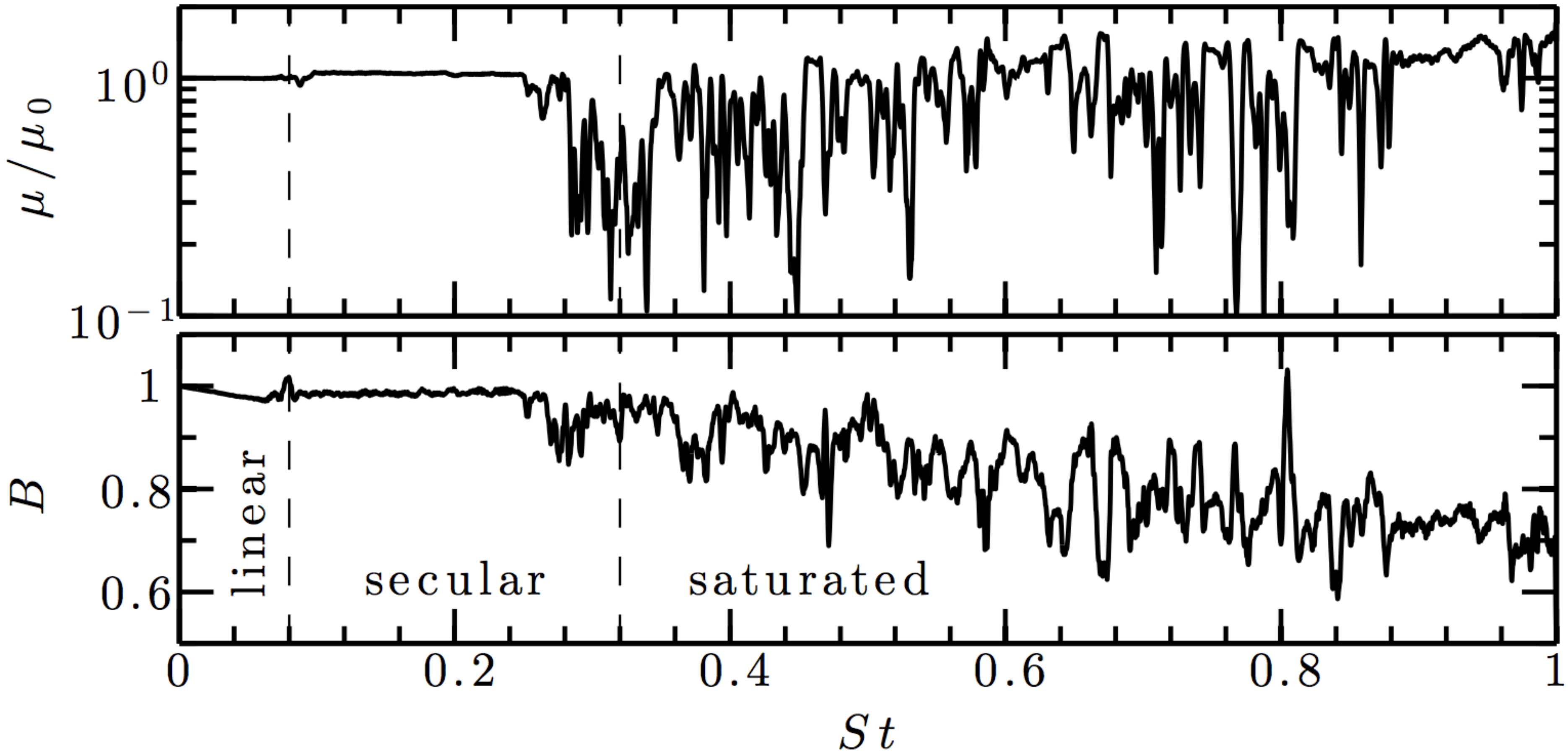






Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Saturated Firehose Scatters Particles



μ conservation is broken at long times, firehose fluctuations scatter particles to maintain pressure anisotropy at marginal level

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Saturated Firehose Scatters Particles

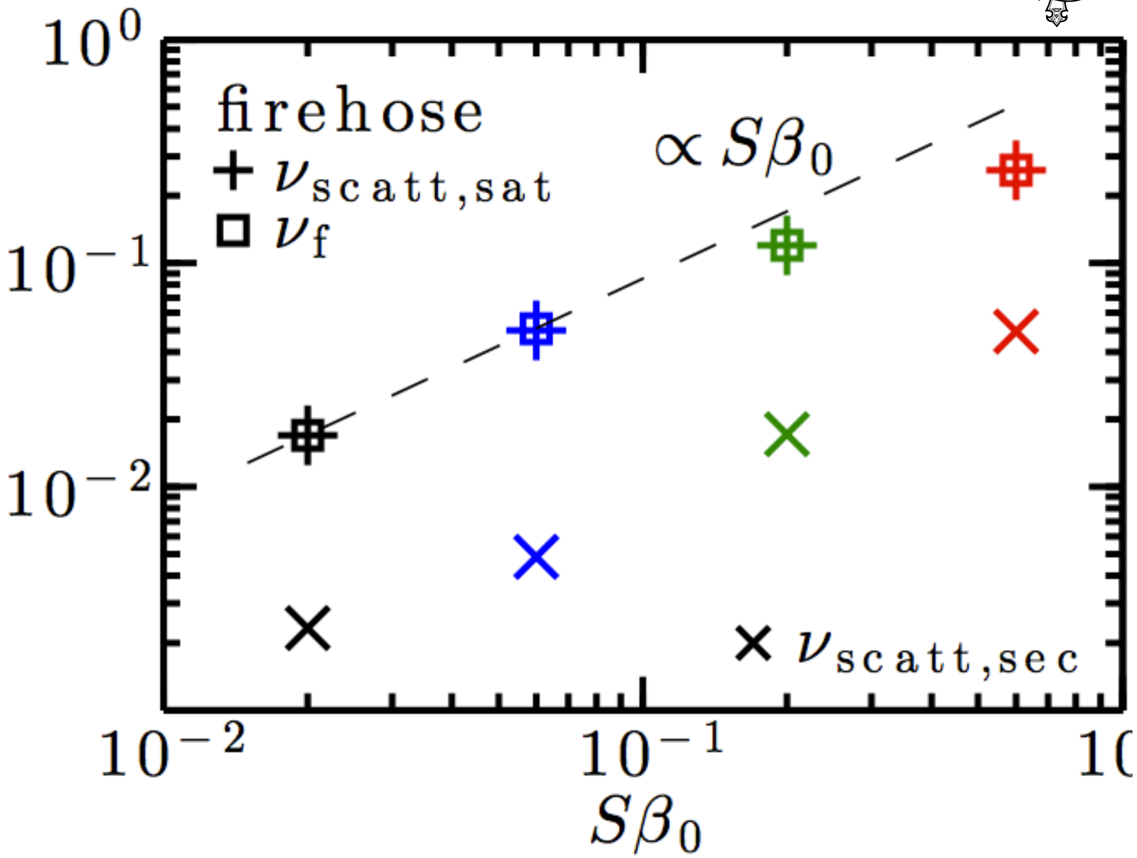
$$\frac{d\Delta}{dt} = 3\frac{d\ln|\langle \mathbf{B}\rangle|}{dt} - \nu_f \Delta$$

$$= 0$$

$$\nu_f = \frac{3}{\Delta}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

$$= -\frac{3\beta}{2}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

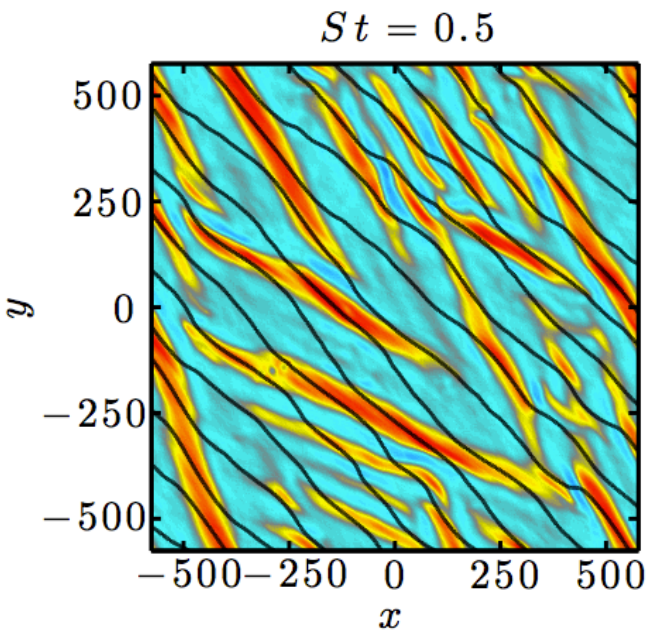
$$\sim S\beta$$

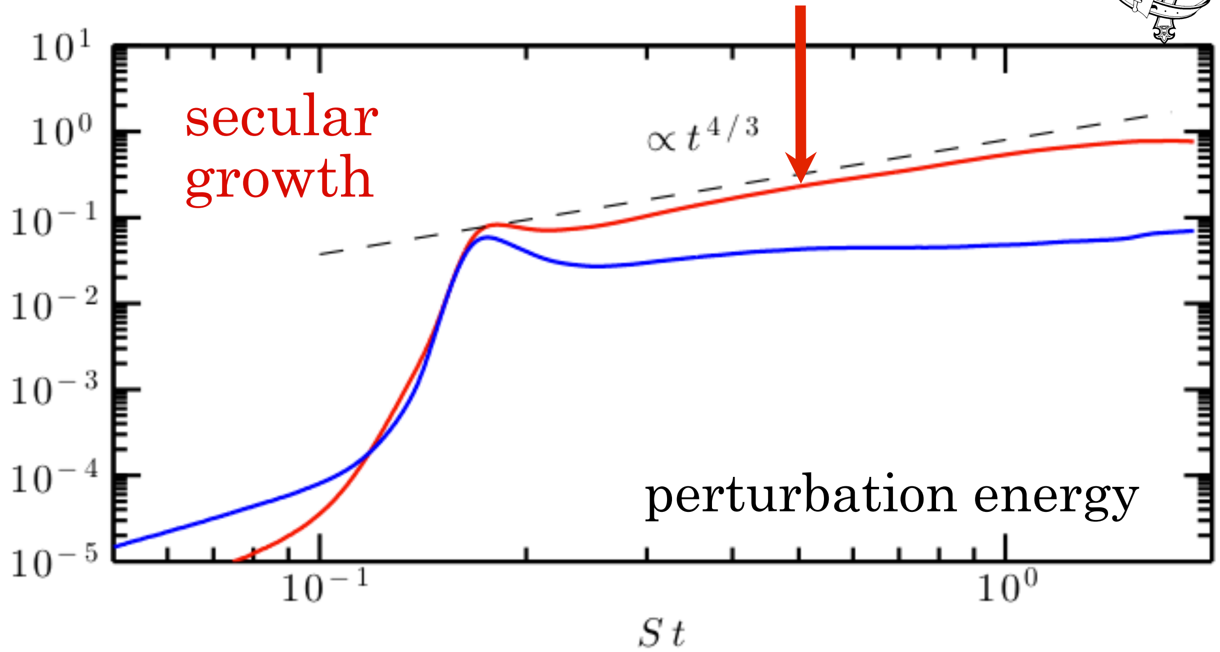


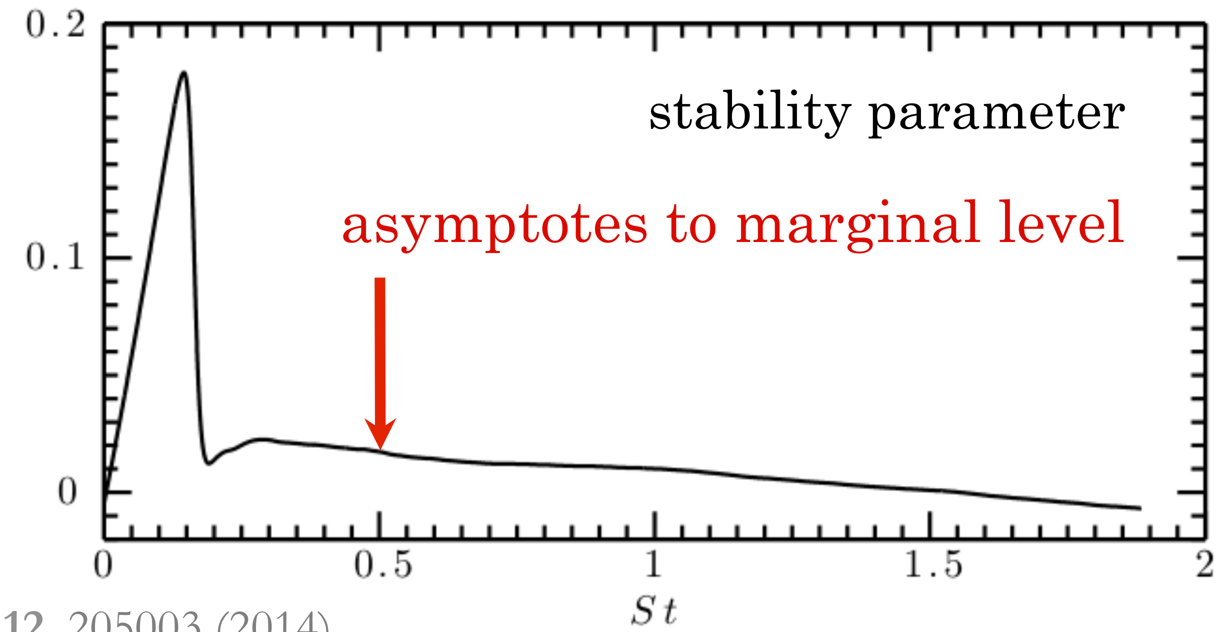
- □ effective collisionality required to maintain marginal stability
- + measured scattering rate during the saturated phase
- × measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Mirror Instability: Secular







Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Mirror Instability: Secular

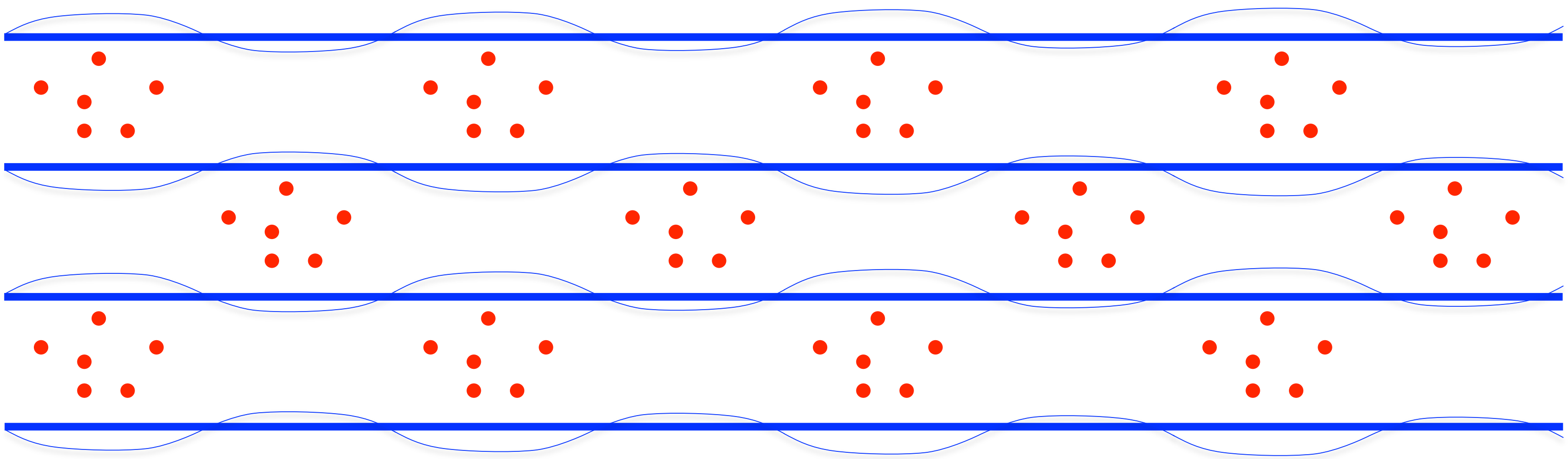
$$\overline{B} = B_0 + \overline{\delta B_\parallel}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{\delta B_\parallel}}{B_0} \right) \to \frac{1}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from mirror-trapped particles in holes (fraction $\sim |\delta B_\parallel / B_0|^{1/2}$)

marginal stability



Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015)

Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

Mirror Instability: Secular

$$\overline{B} = B_0 + \overline{\delta B_\parallel}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} \sim 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t} \overline{\left| \frac{\delta B_\parallel}{B_0} \right|^{3/2}} \right) \to \frac{1}{\beta}$$

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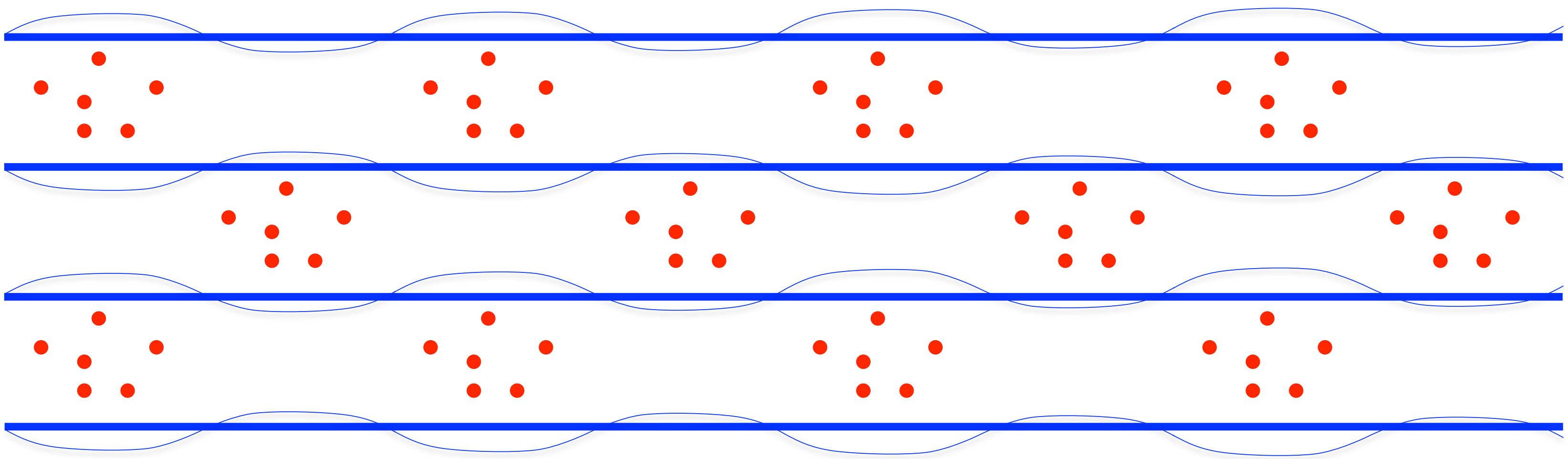

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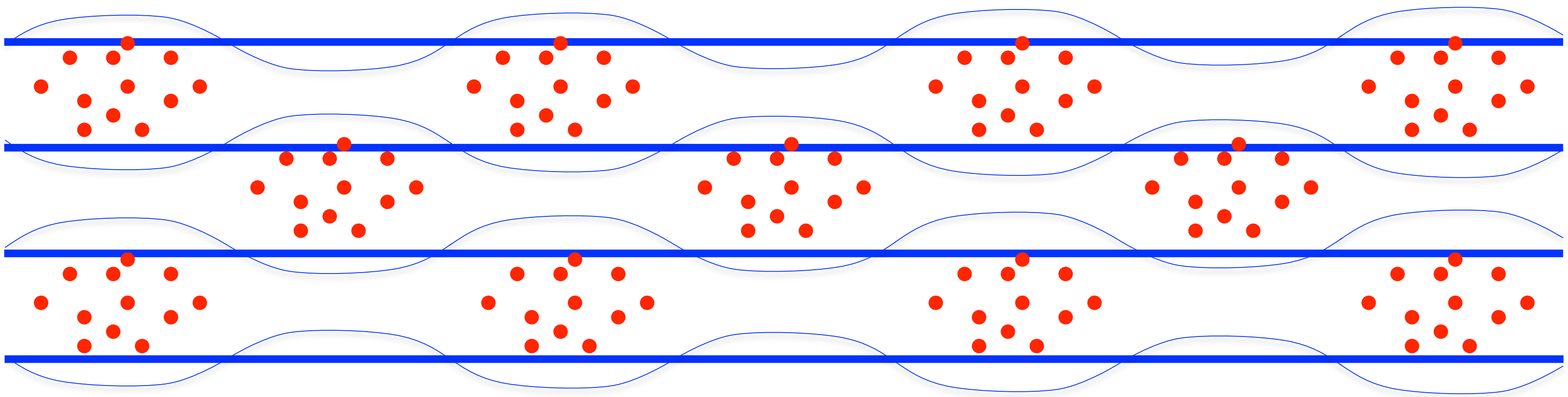
marginal stability



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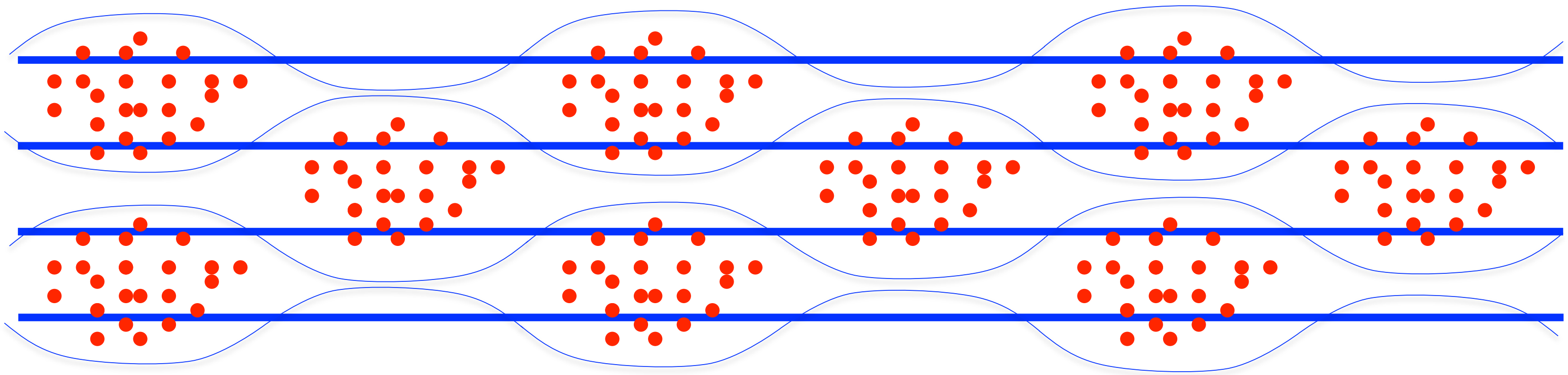
pressure anisotropy driven by shear

pressure anisotropy from mirror-trapped particles in holes (fraction $\sim |\delta B_\parallel / B_0|^{1/2}$)

marginal stability



Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015)
Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

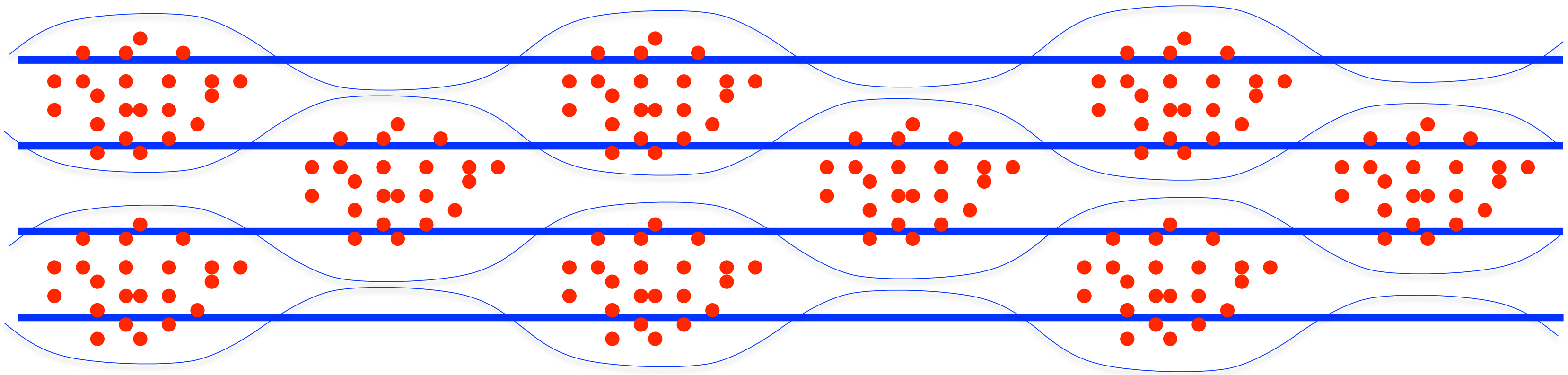
Mirror Instability: Secular

$$\overline{B} = B_0 + \overline{\delta B_\parallel}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} \sim 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}\overline{\left|\frac{\delta B_\parallel}{B_0}\right|^{3/2}} \right) \to \frac{1}{\beta}$$

$$\overline{\left|\frac{\delta B_\parallel}{B_0}\right|^{3/2}} = S\int^t \mathrm{d}t' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{1}{\beta} \quad \Rightarrow \quad \frac{\delta B_\parallel^2}{B_0^2} \sim (St)^{4/3}$$

secular growth

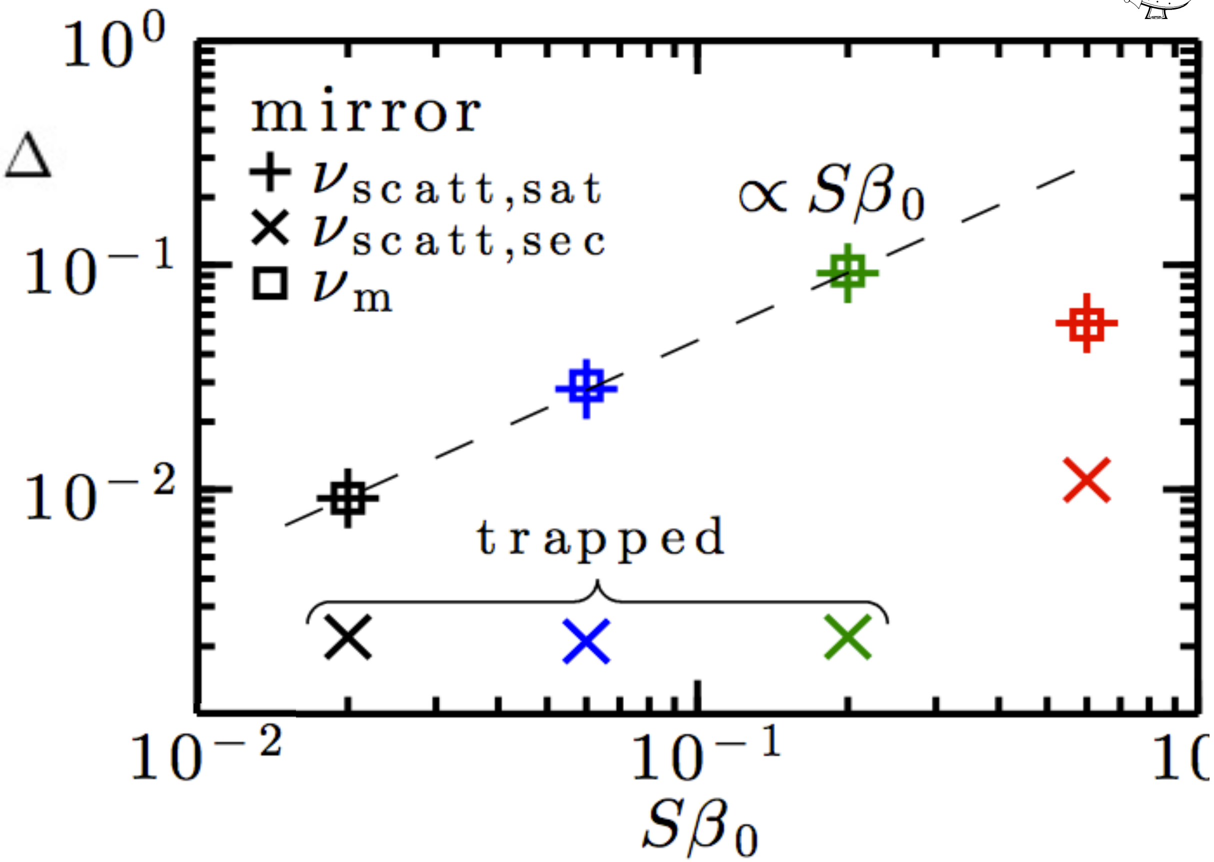


Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015)

Melville, AAS, Kunz, *MNRAS* in press (2016) [arXiv:1512.08131]

Secular Mirror Doesn't Scatter Particles

$$\frac{d\Delta}{dt} = 3\frac{d\ln|\langle \mathbf{B}\rangle|}{dt} - \nu_m \Delta$$
$$= 0$$
$$\nu_m = \frac{3}{\Delta}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$
$$= 3\beta\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$
$$\sim S\beta$$



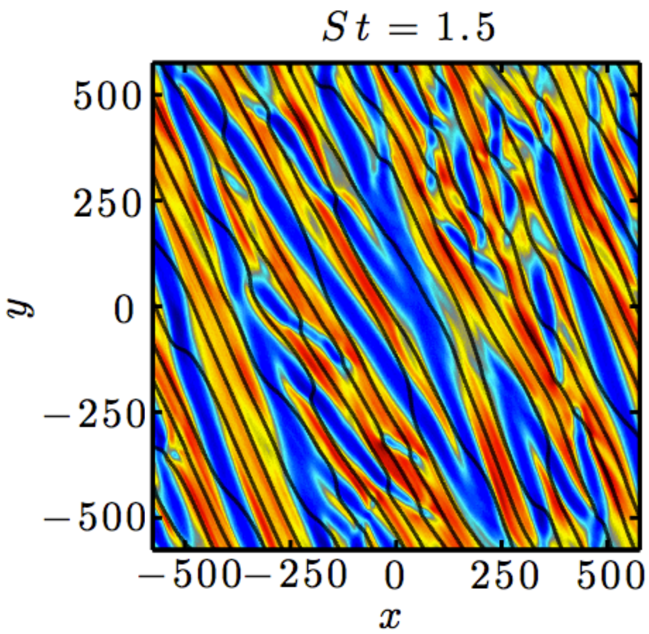
□ effective collisionality required to maintain marginal stability

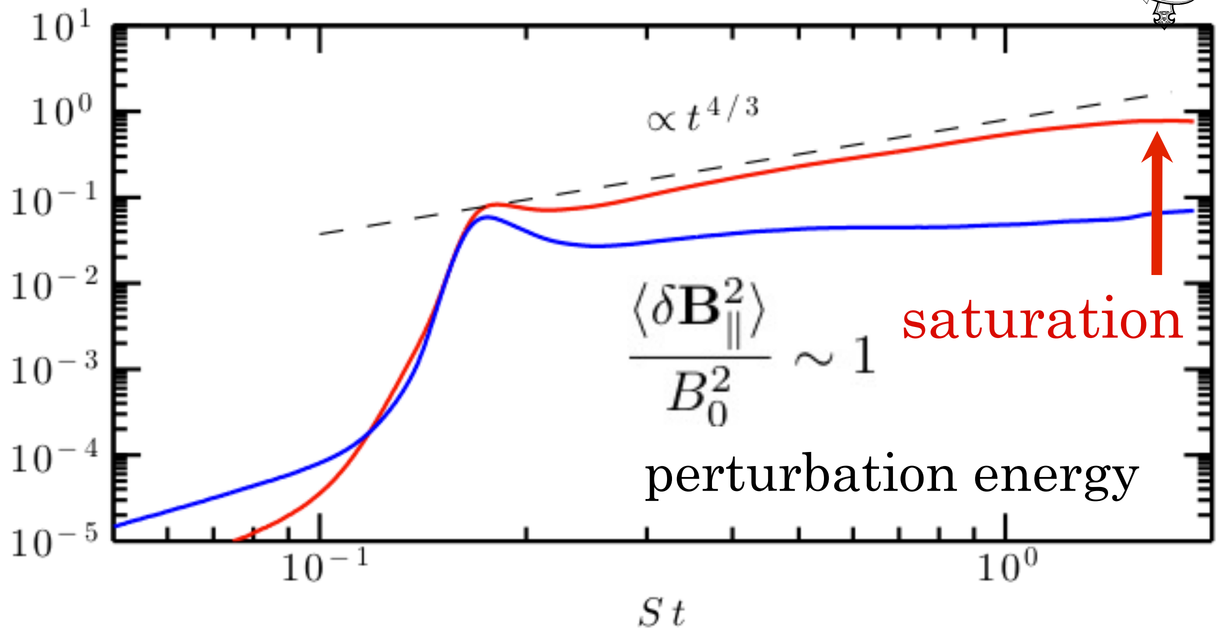
+ measured scattering rate during the saturated phase

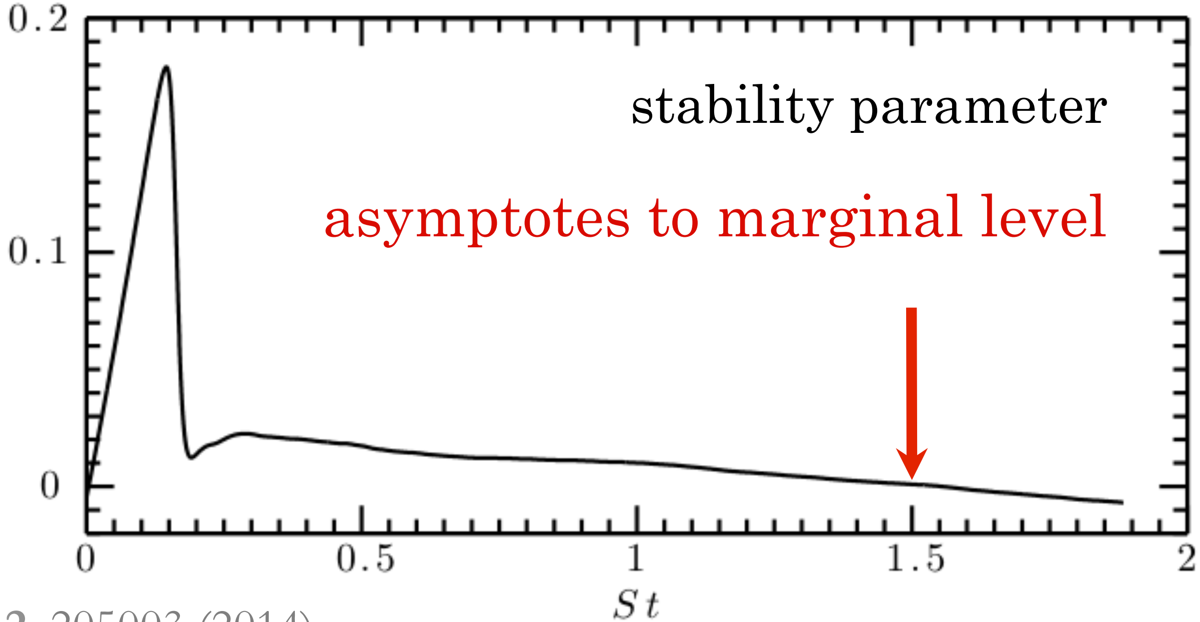
× measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Mirror Instability: Saturated







Kunz, AAS & Stone, *PRL* **112**, 205003 (2014)

Effective Closure Options

Option I: Suppress stretching

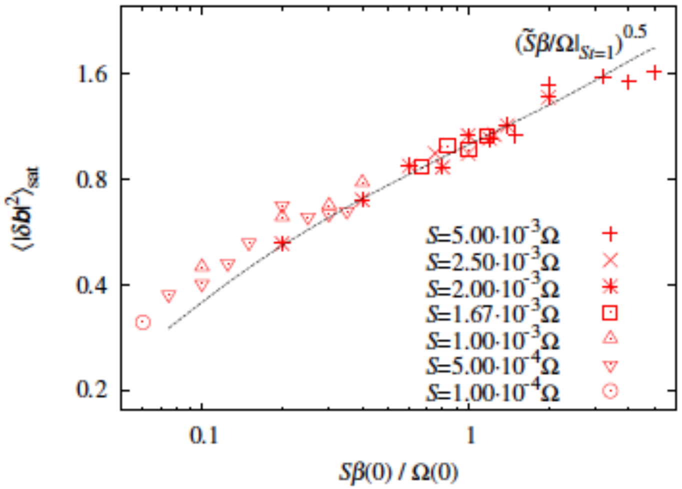
$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

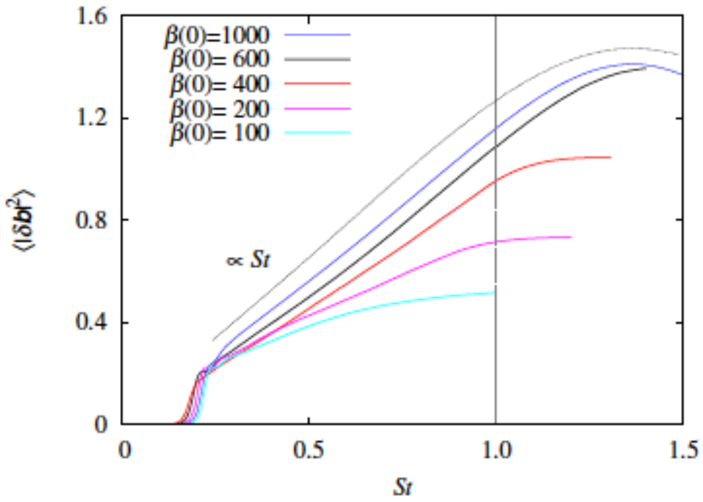
Option II: Enhance collisionality

This happens for firehoses (also mirrors in saturation & decaying)

Melville, AAS & Kunz
arXiv:1512.08131

Effective Closure Options





This in fact also happens for firehoses, at ultra-high beta $\beta > \Omega/S$

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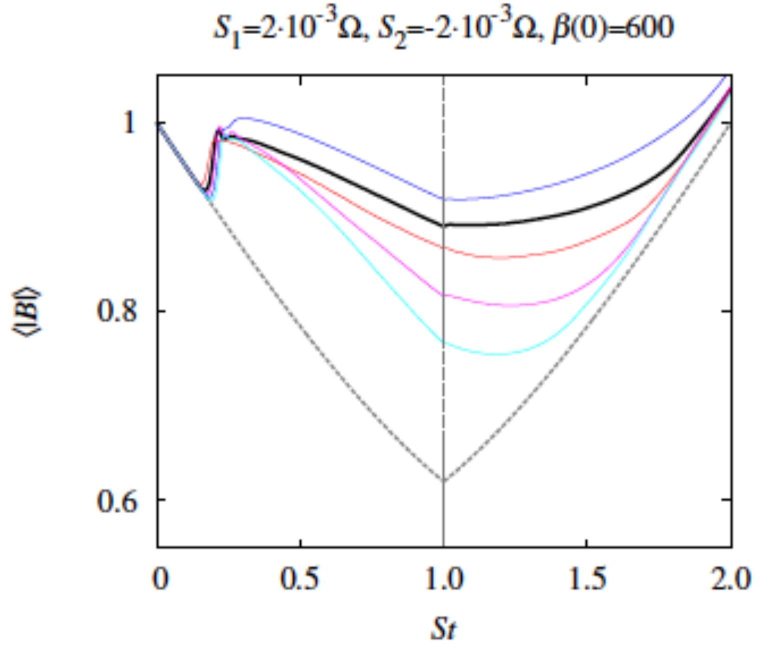

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This happens for firehoses (also mirrors in saturation & decaying)

Option II: Enhance collisionality

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Effective Closure Options

Option III: Skew the average towards regions of weaker field by trapping particles

This happens for secularly growing mirrors

This in fact also happens for fireboses, at ultra-high beta $\beta > \Omega/S$

Option I: Suppress stretching

$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\overline{\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t}} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

This happens for fireboses (also mirrors in saturation & decaying)

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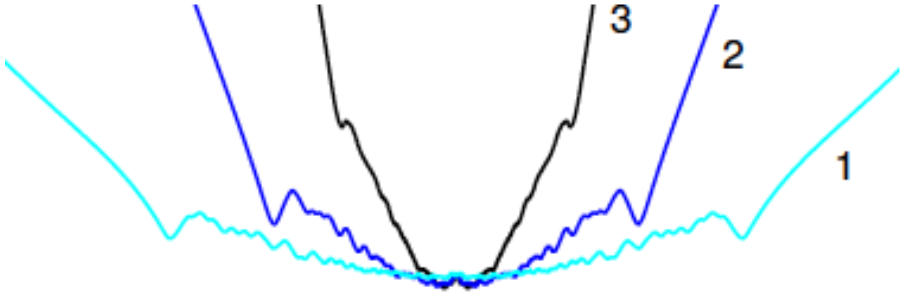
Theoretical Plasma Dynamo: UNSOLVED

- In firehose regions, anomalous scattering will marginalise anisotropy



Melville, AAS & Kunz
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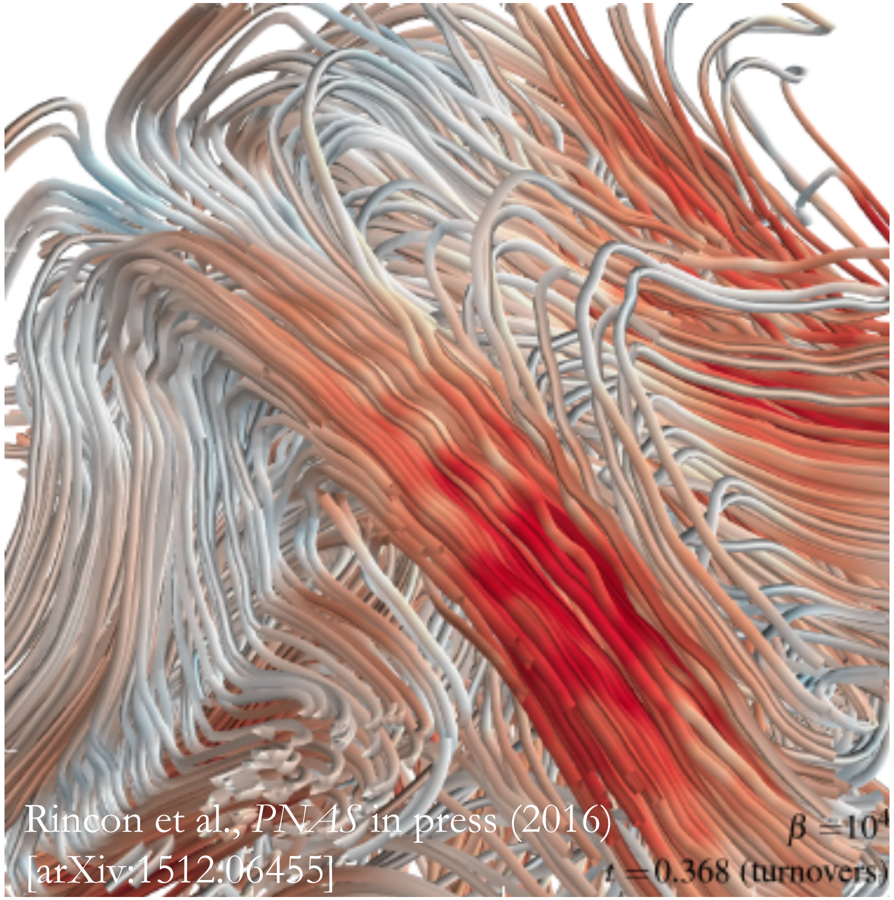
- It appears that in the stretching regions, B can grow, at the price of "mirror-bubble" infestation (as long as the stretching does not last longer than ~ turnover time



Theoretical Plasma Dynamo: UNSOLVED

- In firehose regions, anomalous scattering will marginalise anisotropy

Eff. collisionality: $\nu_{\text{eff}} \sim S\beta$

to keep anisotropy marginal:

$$\Delta \sim \frac{S}{\nu_{\text{eff}}} \sim \beta^{-1}$$

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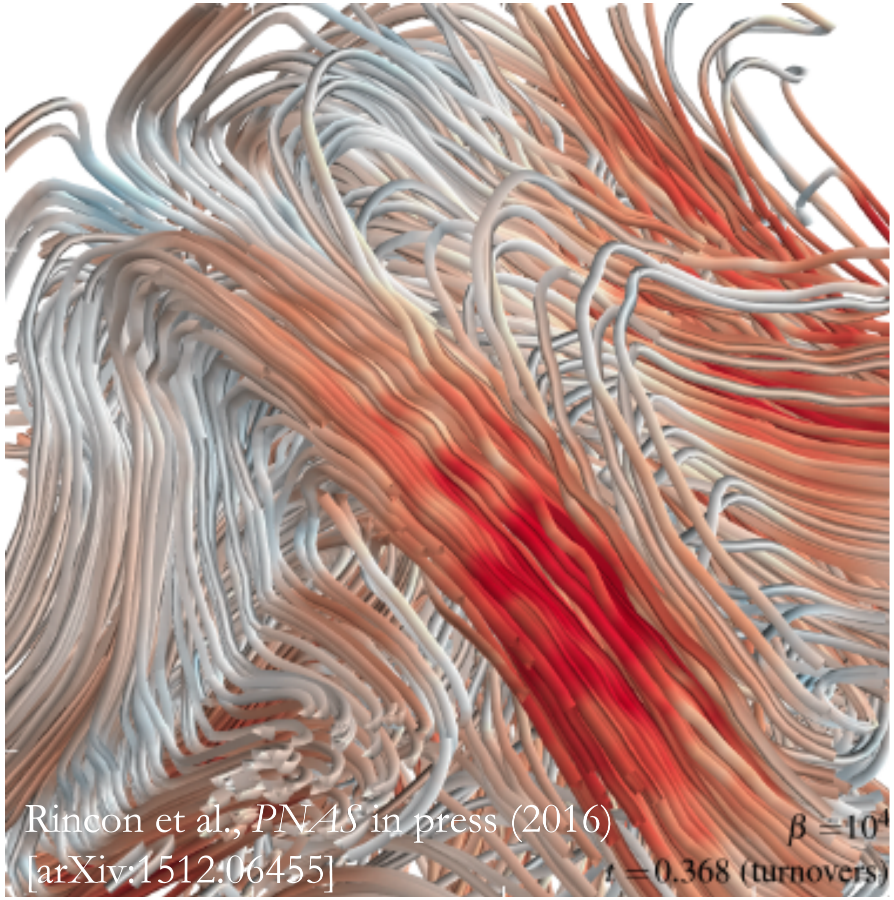

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Theoretical Plasma Dynamo: UNSOLVED

- In firehose regions, anomalous scattering will marginalise anisotropy

Eff. collisionality: $\nu_{\rm eff} \sim S\beta$

Eff. viscosity: $\mu_{\rm eff} \sim \dfrac{p}{\nu_{\rm eff}} \sim \dfrac{B^2/8\pi}{S}$

NB: regions of weaker field are less viscous!

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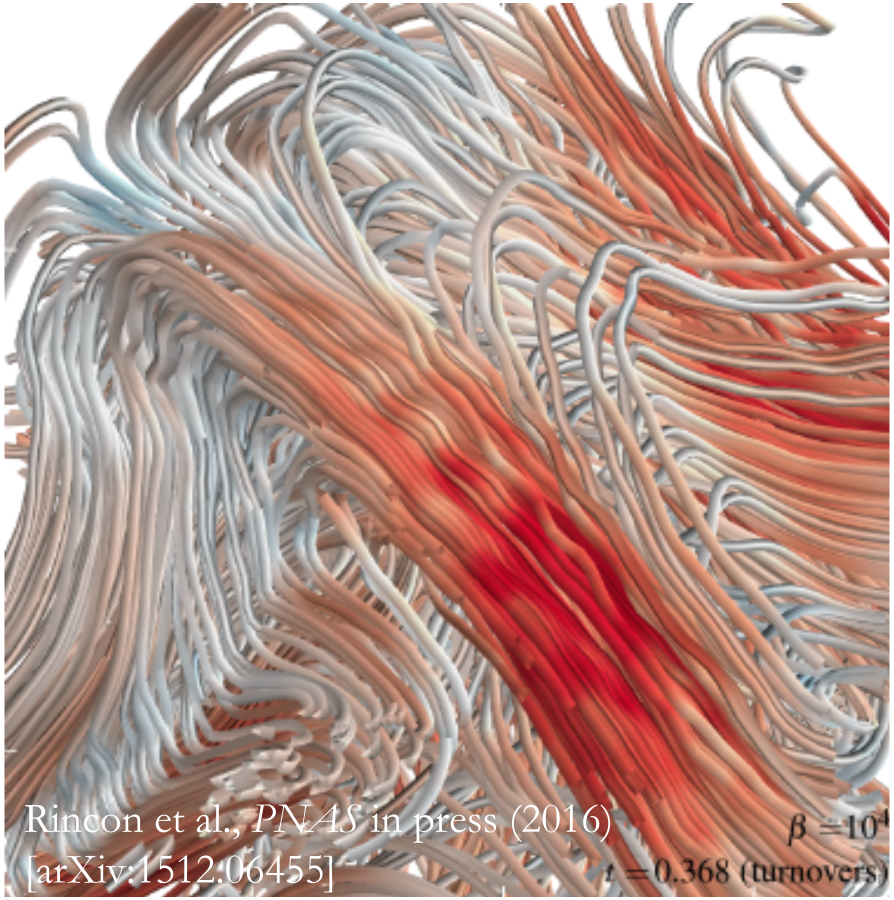

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Energy flux (Kolmogorov): $\varepsilon \sim \dfrac{\rho U^3}{L}$

Fastest turnover rate:

$$S \sim \left(\frac{\varepsilon}{\mu_{\rm eff}}\right)^{1/2} \sim \mathrm{Ma}\left(\frac{U}{L}\nu_{\rm eff}\right)^{1/2} \sim \frac{U}{L}\,\mathrm{Ma}^2\beta$$

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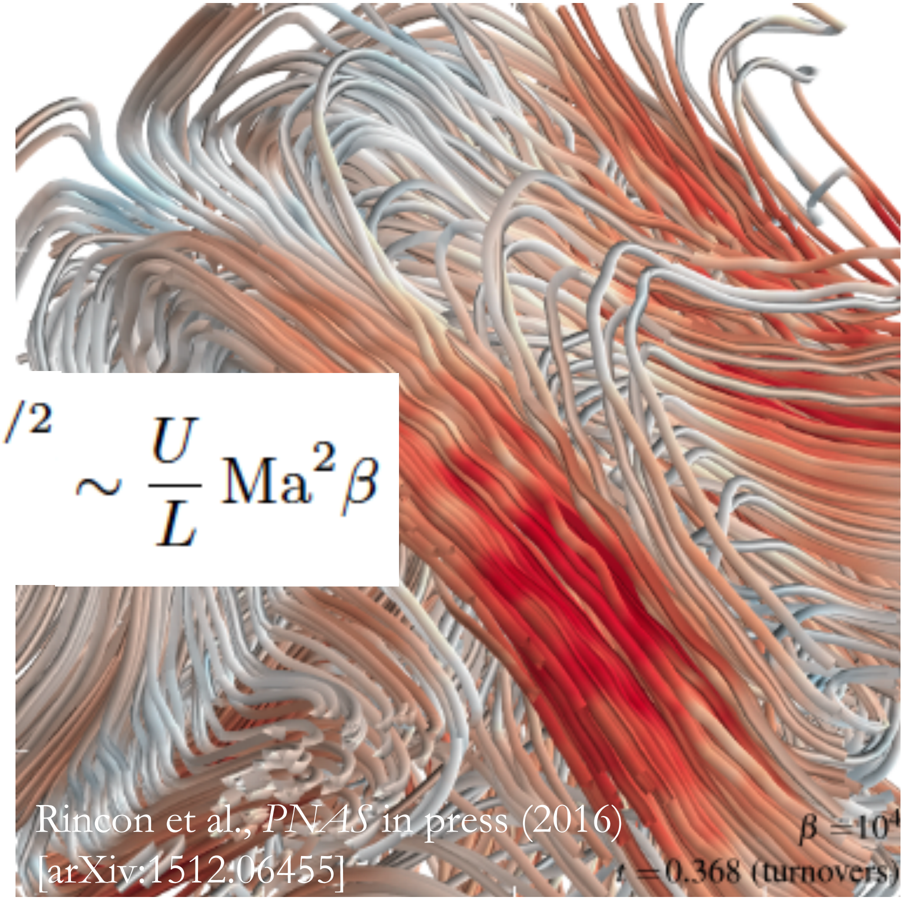

Rincon et al., *PNAS* in press (2016) [arXiv:1512.06455]

Melville, AAS & Kunz arXiv:1512.08131

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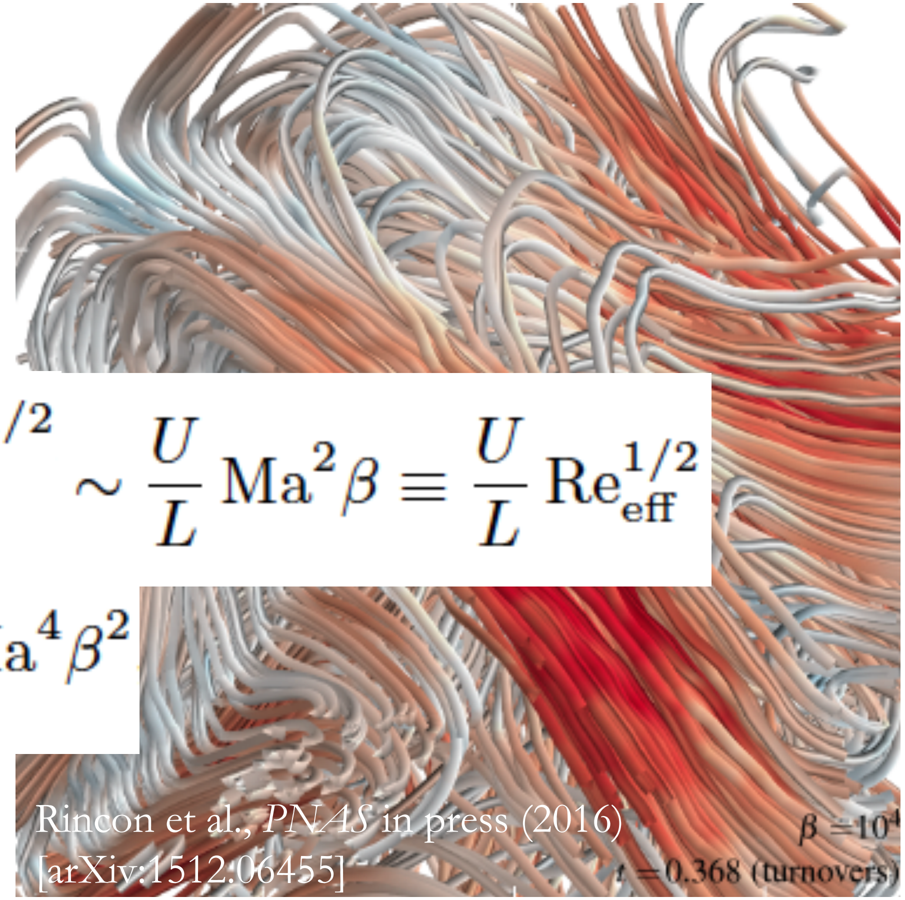
Effective Reynolds number: $\mathrm{Re}_{\rm eff} = \mathrm{Ma}^4\beta^2$

- Regions of decreasing field will quickly break up?

- It appears that in the stretching regions, B can grow, at the price of "mirror-bubble" infestation (as long as the stretching does not last longer than ~ turnover time



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NB: faster if the field is weaker

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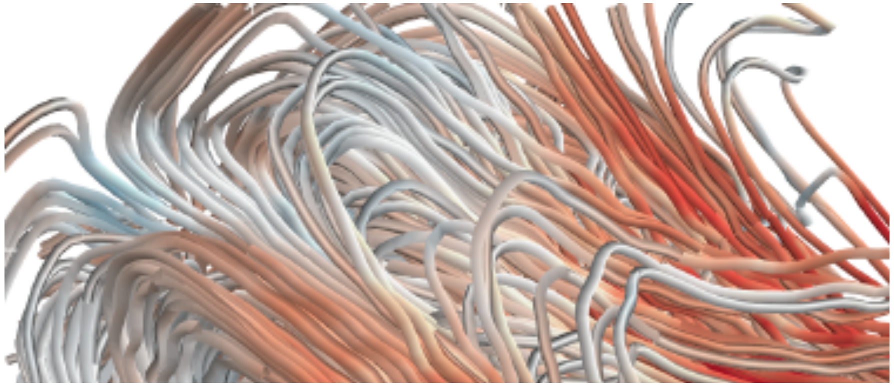

So magnetic energy grows in one turnover time from any level:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{B^2}{8\pi} \sim S\,\frac{B^2}{8\pi} \sim \varepsilon$$

(but only if this enhanced collisionality persists)

Mogavero & AAS, *MNRAS* **440**, 3226 (2014)

Theoretical Plasma Dynamo: UNSOLVED

- In firehose regions, anomalous scattering will marginalise anisotropy
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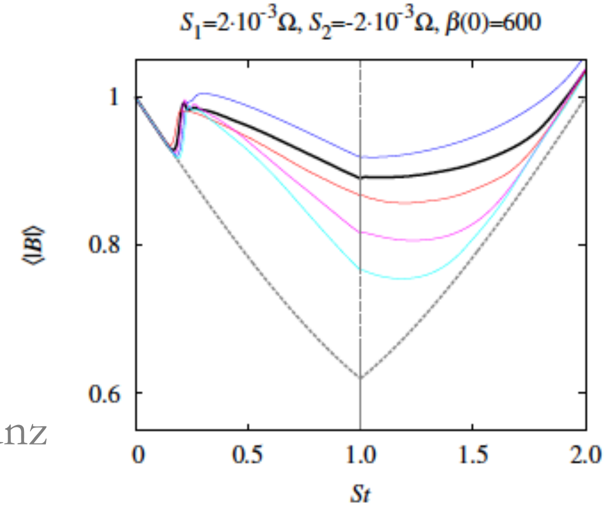
Thus, it is quite difficult to decrease B in a weakly collisional plasma, while growth is OK (and maybe even faster!).
Good news for fast plasma dynamo!

…especially at high beta

$$\beta > \Omega/S$$



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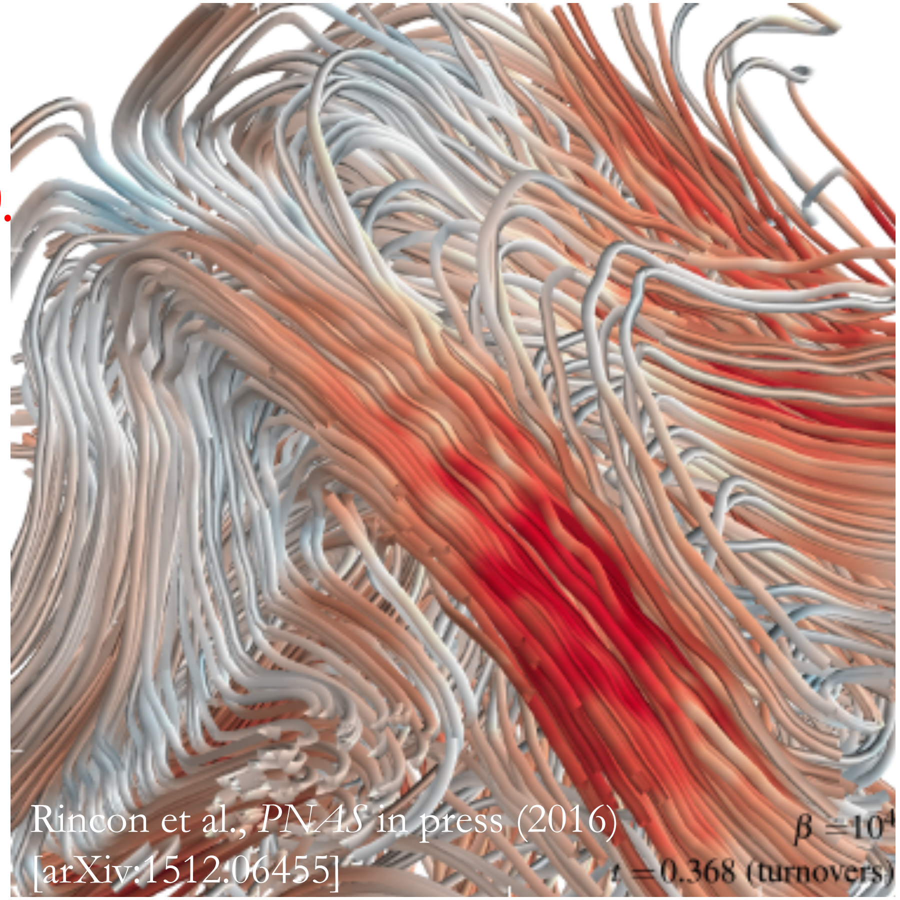

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Upcoming historic events:

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(F. Rincon; M. Kunz & D. St-Onge)

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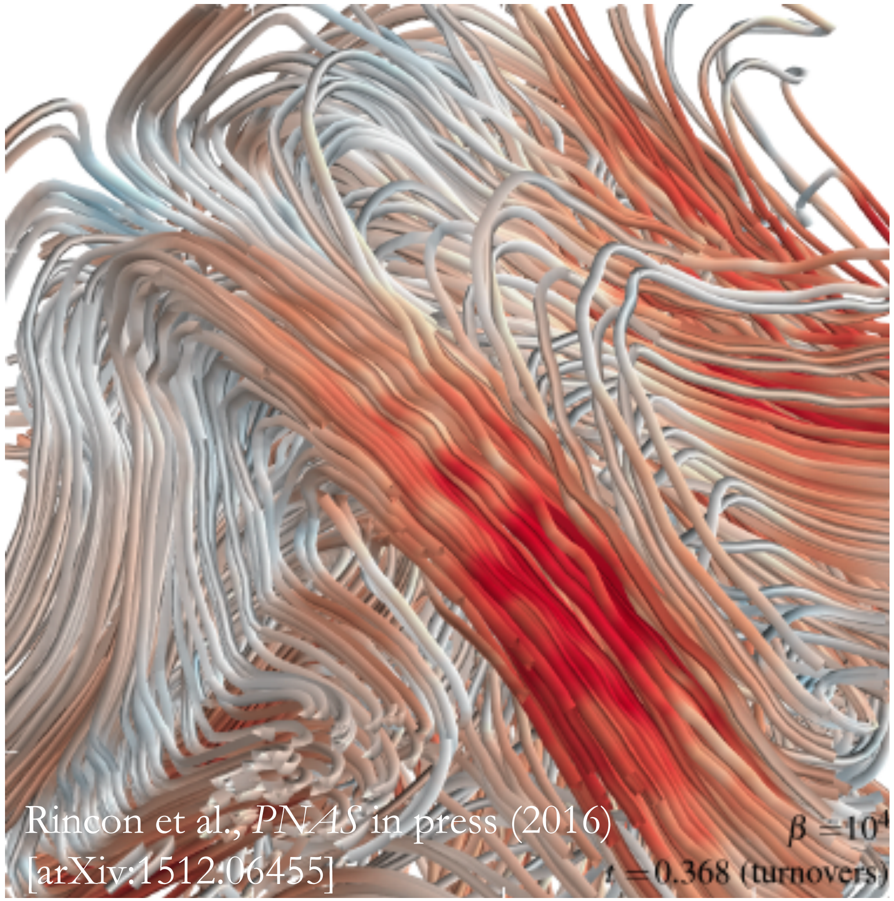

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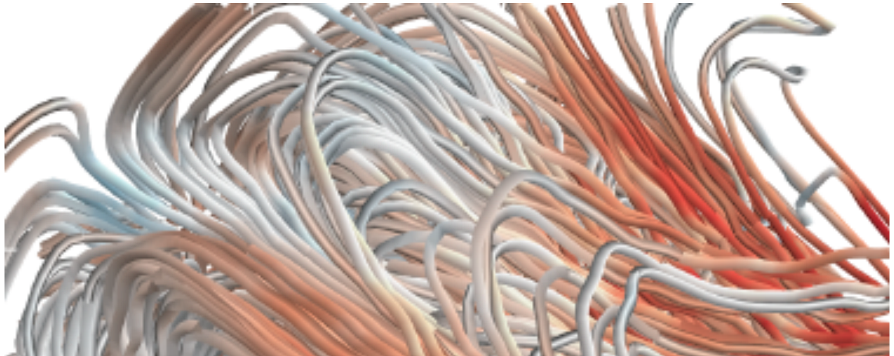

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At any rate, high-beta, weakly collisional plasma dynamics are interesting and still to be understood.

For astrophysics, paradigm change in the air?

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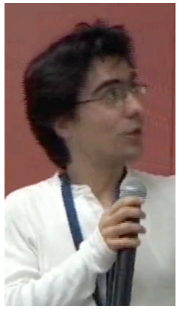

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