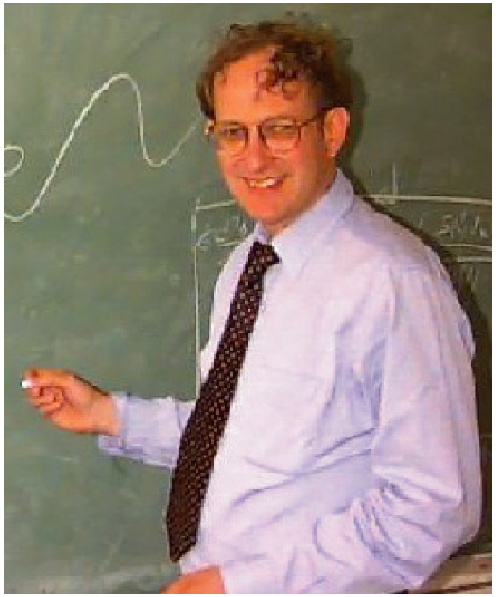
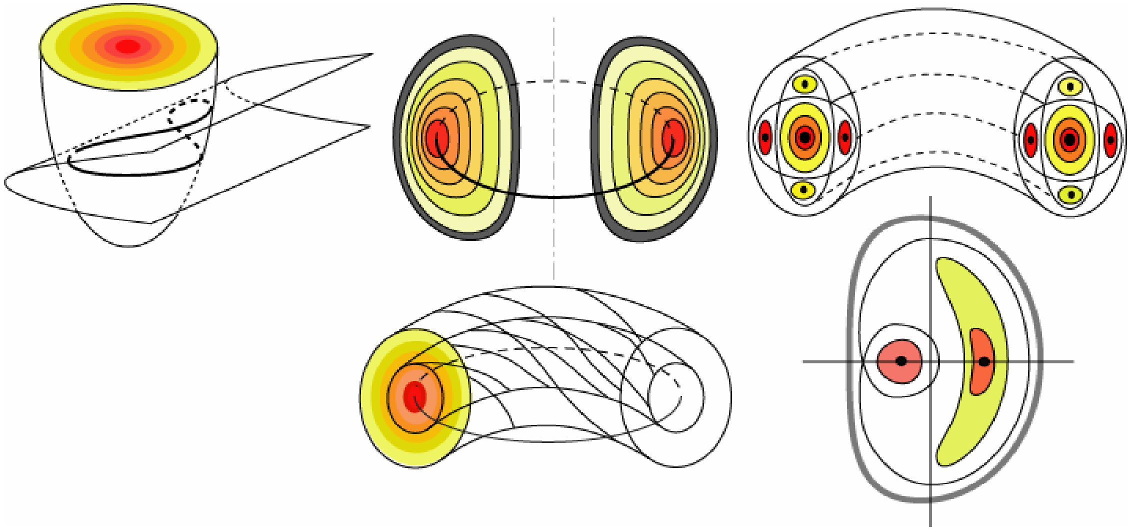
Solved and Unsolved Problems in Plasma Physics

A symposium in honor of Nat Fisch

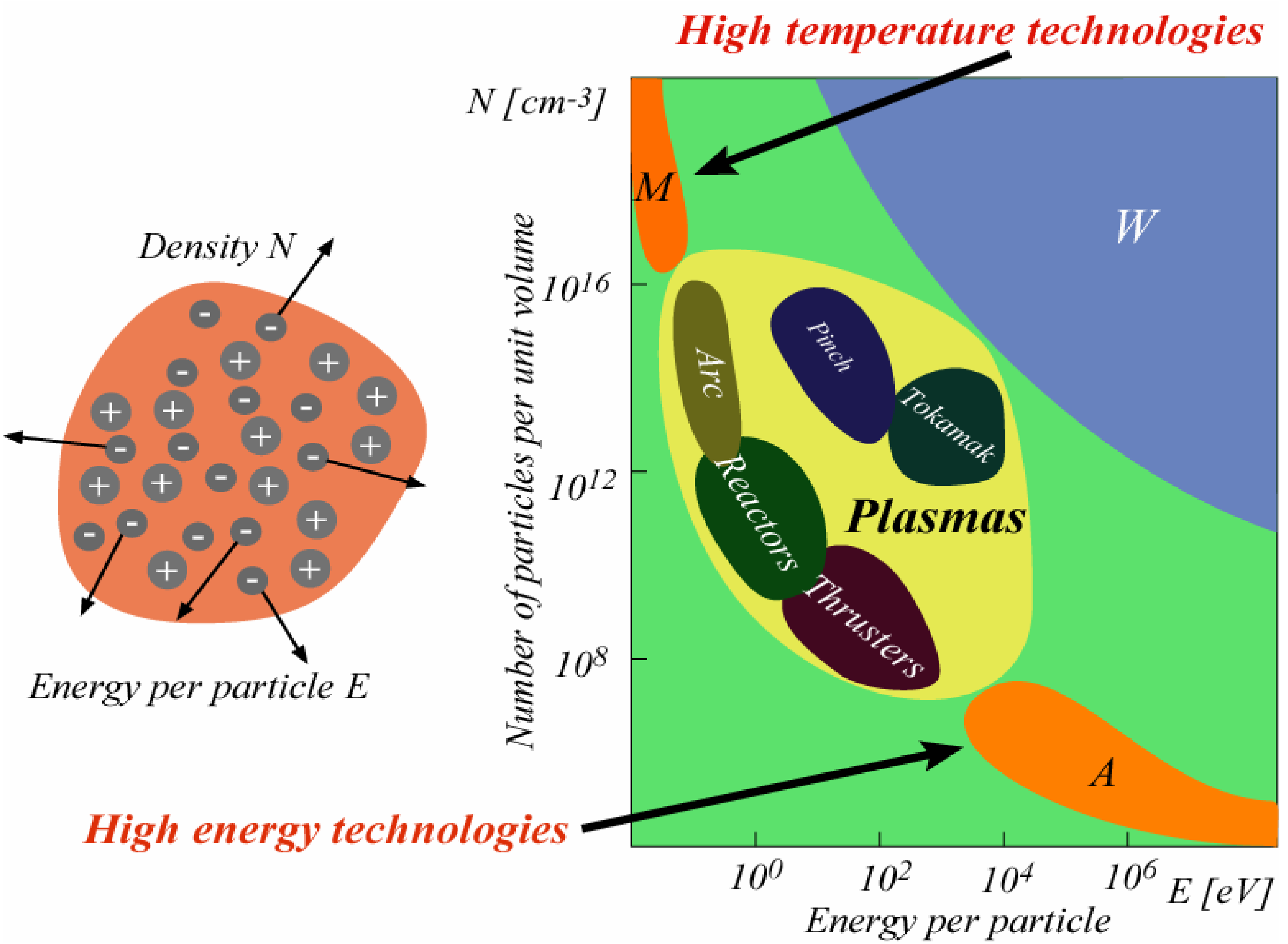




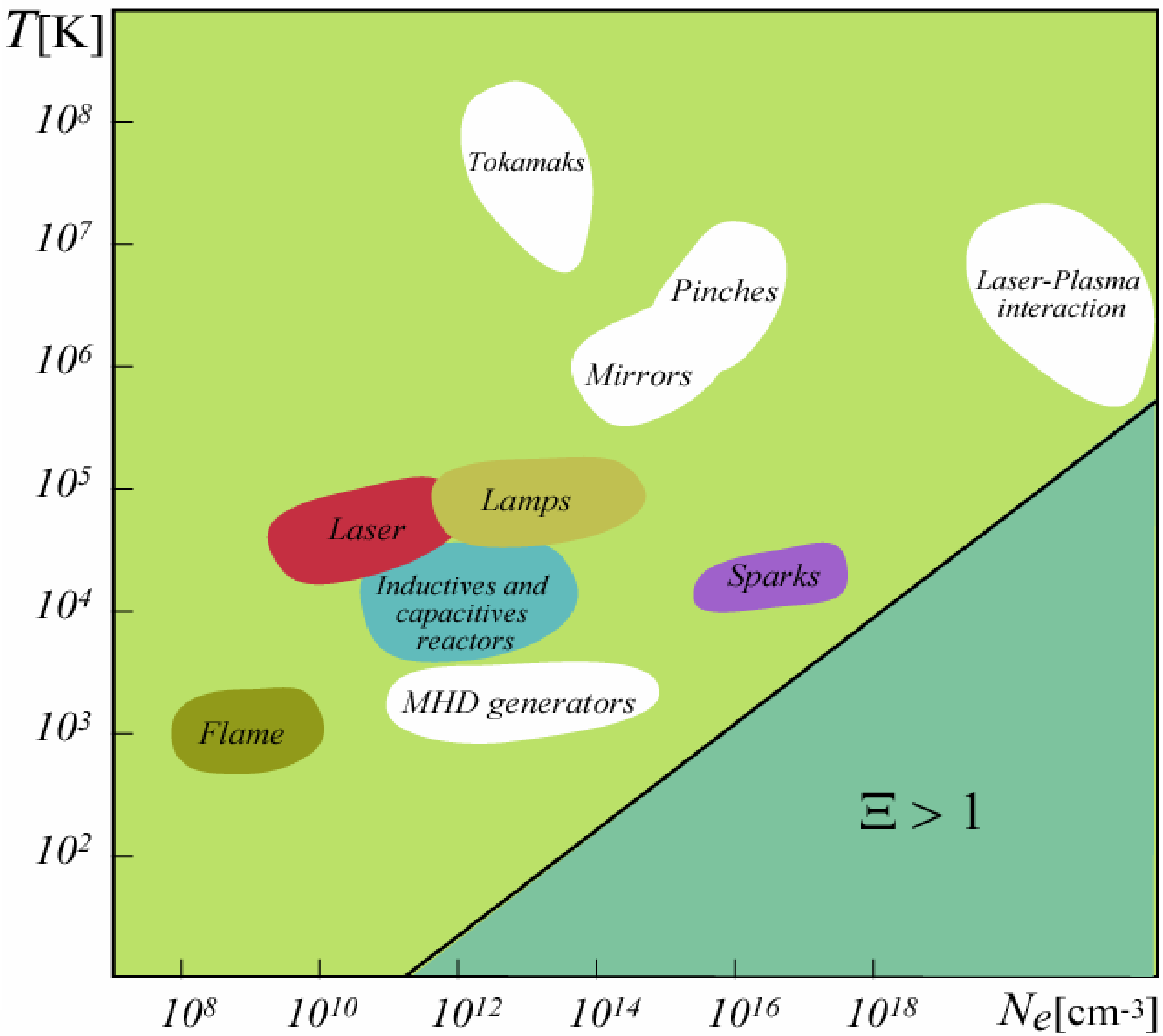
Rotations and Instabilities for Isotope and Mass Separations

JM Rax
Université de Paris 11 - Ecole Polytechnique

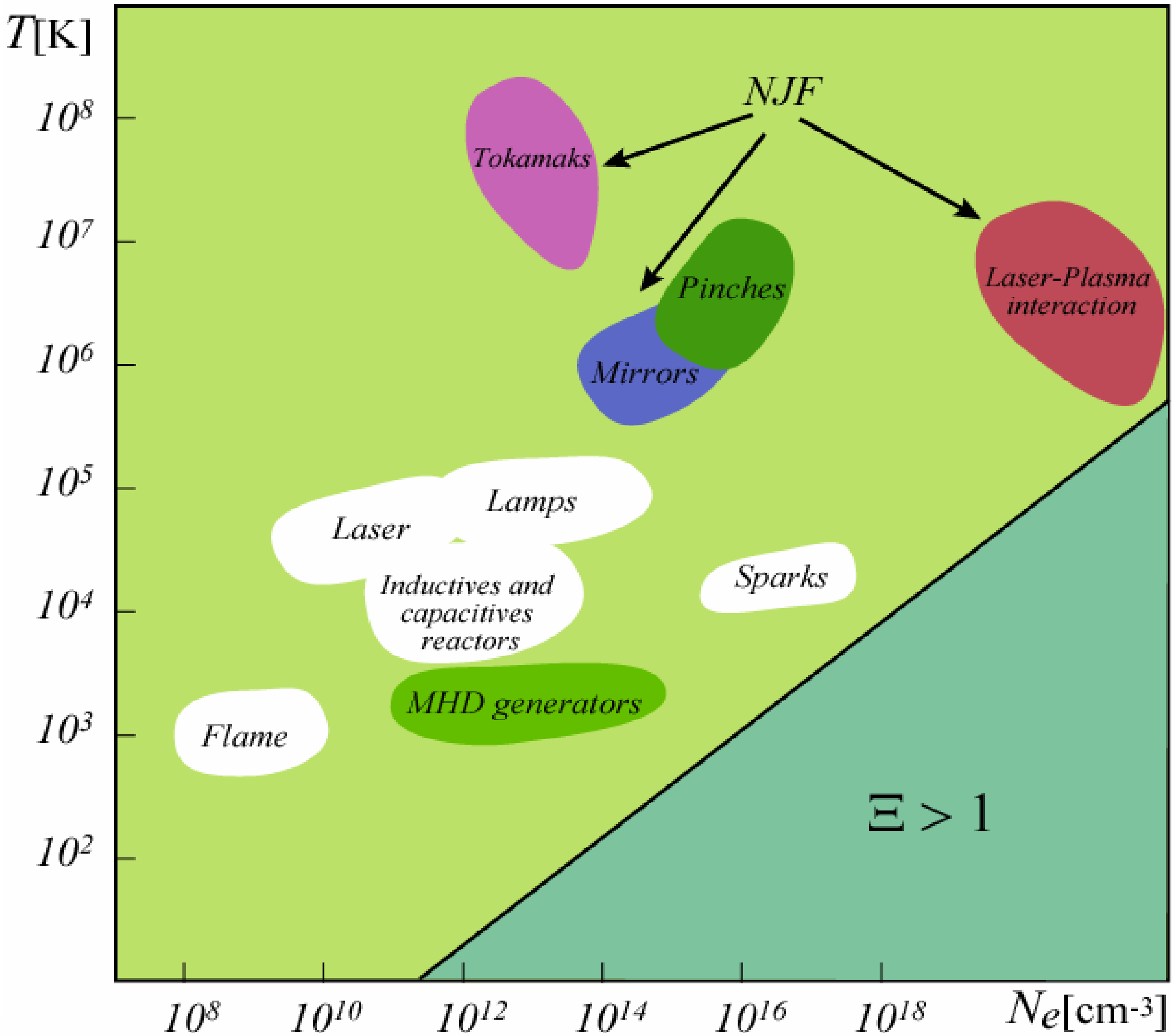
Plasmas problems and Plasmas devices



Solved problems in plasma physics



Unsolved problems in plasma physics



Rotations and instabilities for isotope and mass separation

Gueroult, R., Hobbs, D.T. & Fisch, N.J. 2015 Plasma filtering techniques for nuclear waste remediation. *Journal of Hazardous Materials* **297**, 153-159.

Fission Energy $^{235}U \longrightarrow$ **200 MeV**

Plasma Processing $^{235}U\ ^{238}U\ldots \longrightarrow$ << **10 keV**

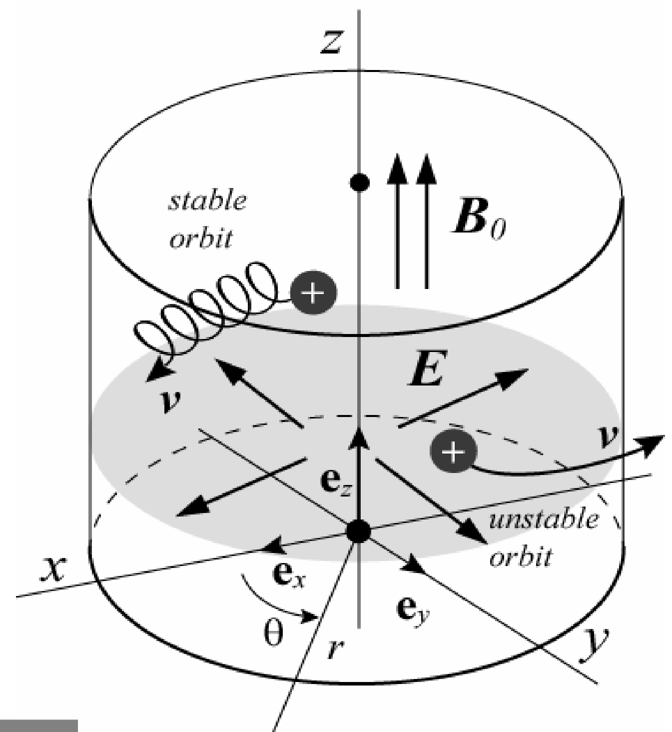
$$dN/dt = n\, C_s\, S\, T \longrightarrow 10 \text{ T/Year}$$

1% $\longrightarrow$ **50-60**

$$\boxed{\frac{q}{m}\left[\mathbf{E}\left(x,y\right),\mathbf{B}_0\right] = \left[\Omega_E^2 x\ \mathbf{e}_x + \Omega_E^2 y\ \mathbf{e}_y, \Omega_c\ \mathbf{e}_z\right]}$$

Brillouin condition

$$\boxed{\Omega_c^2 > 2\Omega_E^2}$$

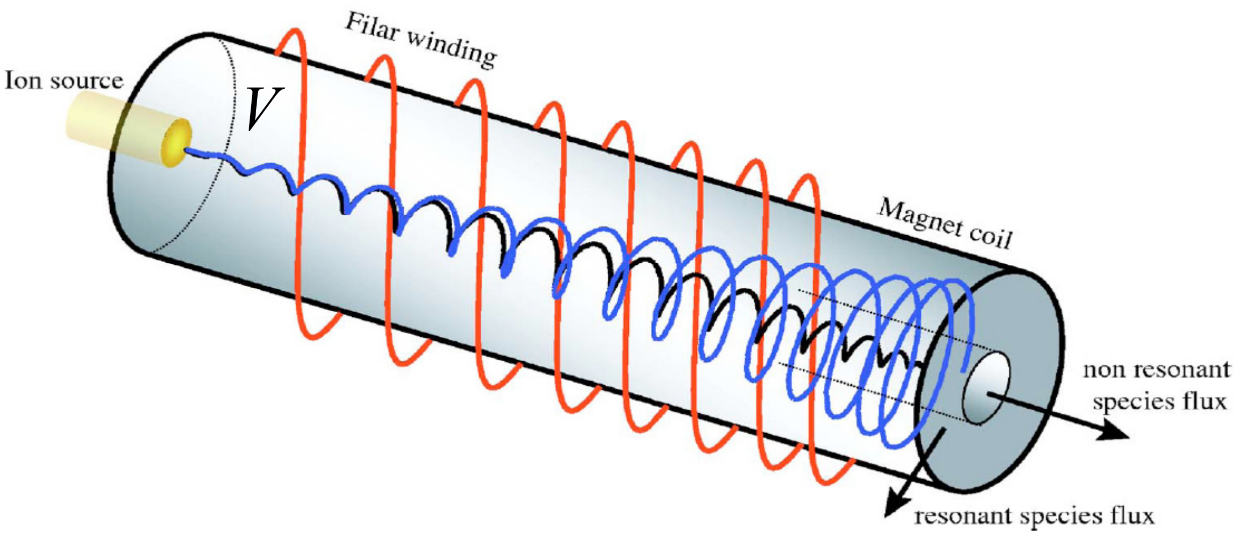


Rax, J.M., Fruchtman, A., Gueroult, R. & Fisch, N.J. 2015 Breakdown of the Brillouin limit and classical fluxes in rotating collisional plasmas. *Phys. Plasmas* **14**, 092101-1-092101-12.

Gueroult, R., Rax, J.M. & Fisch, N.J. 2014 The double well mass filter. *Phys. Plasmas* **21**, 020701-1-020701-3.

Autoresonant separation with an helical tapered wigler

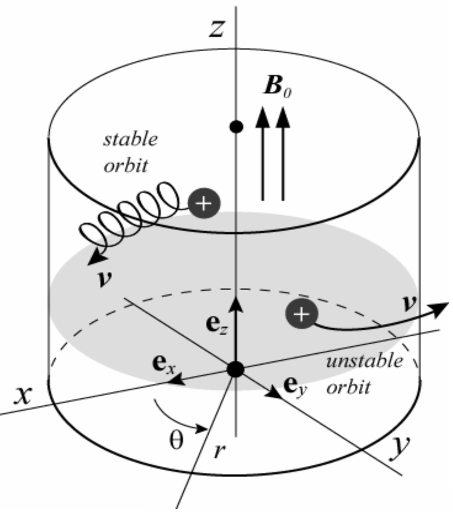
$$\frac{q}{m}\mathbf{A}(x,y,z)=\left[\frac{q}{m}A\cos\int_0^z k(u)\,du-\frac{\Omega_c}{2}y\right]\mathbf{e}_x+\left[\frac{q}{m}A\sin\int_0^z k(u)\,du+\frac{\Omega_c}{2}x\right]\mathbf{e}_y$$



$$\boxed{k(z)V=\Omega_c\left[\cos\left(\frac{2qA\Omega_c}{mV^2}z+\sum_{n=1}^{+\infty}\frac{J_n(n)}{n}\sin\left(\frac{4nqA\Omega_c}{mV^2}z-\frac{n\pi}{2}\right)\right)\right]^{-1}}$$

Rax, J.M., Robiche, J. & Fisch, N.J. 2007 Autoresonant ion cyclotron isotope separation. *Phys. Plasmas* **14**, 043102-1-43102-8.

3 principles for plasma rotation



(1) ***E*** *(radial)* cross ***B*** *(axial)* drift *(azimuthal)*

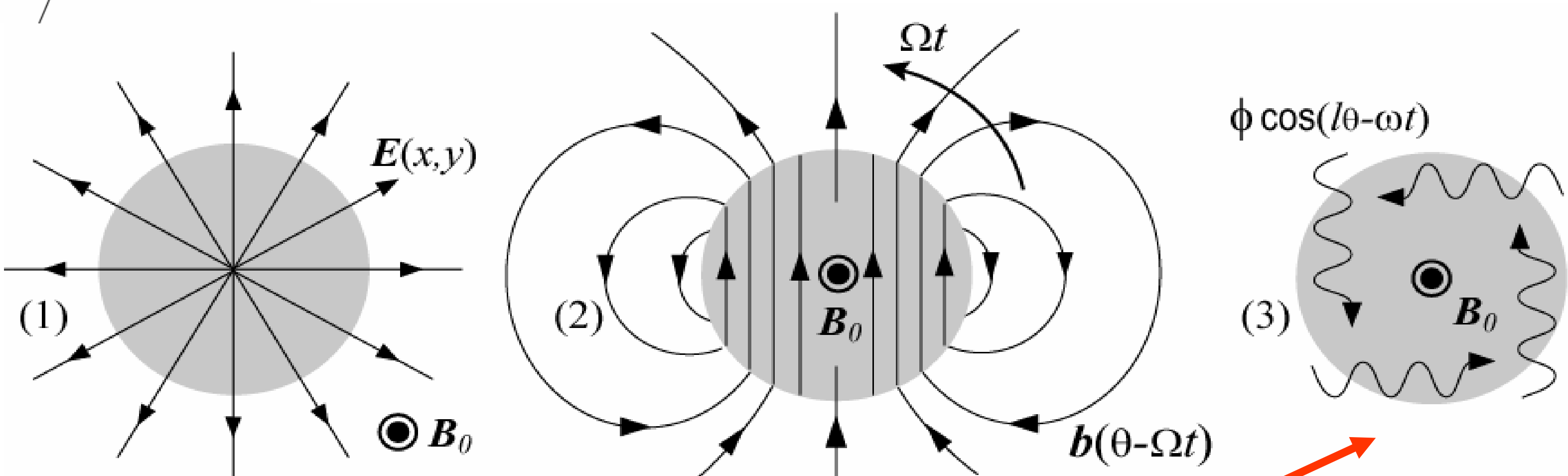
(2) *Frozen in Law* with rotating ***b***(*t*) field

(3) *Direct injection of angular momentum* with waves

$$\Phi\left(x,y,z,t\right)=\phi\left(r\right)\cos\left(l\theta+kz-\omega t\right)$$

$$\boxed{\frac{\delta \mathbf{L}}{\delta W}=\frac{l}{\omega}\mathbf{e}_z}$$

$$\boxed{kV=n\Omega_c+\omega}$$

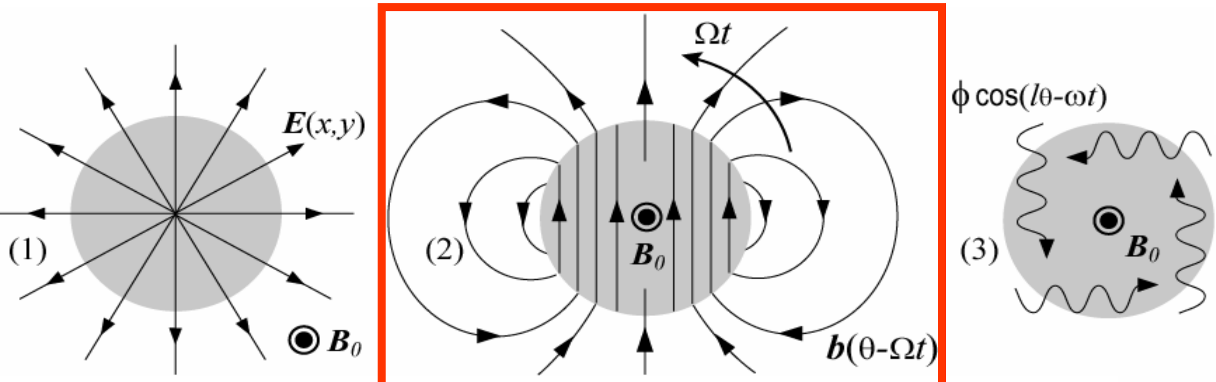


Fetterman, A.J. & Fisch, N.J. 2011 Wave-particle interactions in rotating mirrors. *Phys. Plasmas* **18**, 055704-1-055704-4.

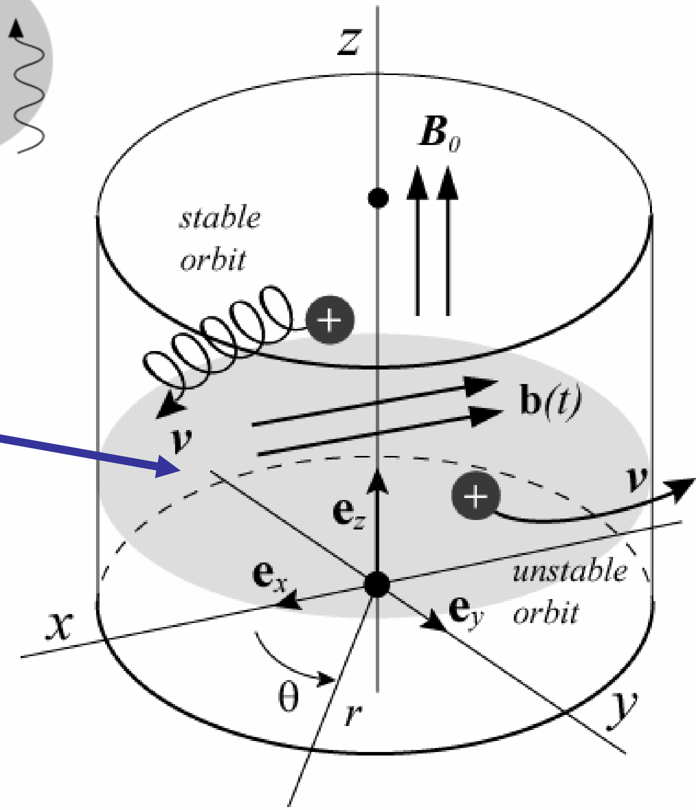
Fetterman, A.J. & Fisch, N.J. 2009 Wave-driven countercurrent plasma centrifuge. *Plasma Sources Science and Technology* **18**, 045003.

Fetterman, A.J. & Fisch, N.J. 2011 The magnetic centrifugal mass filter. *Phys. Plasmas* **18**, 094503.

3 basics rotating fields



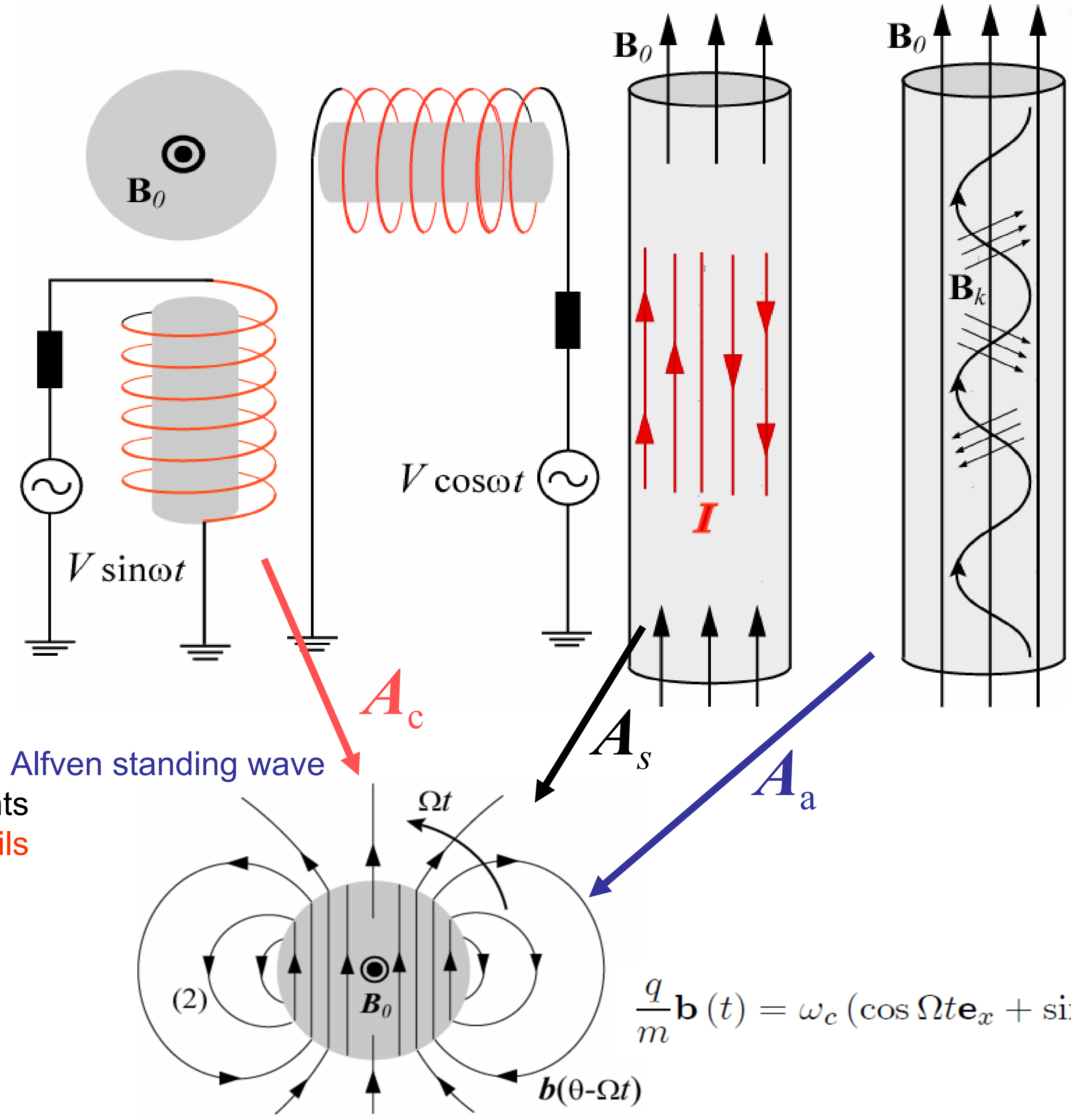
$$\frac{q}{m}\mathbf{b}\left(t\right)=\omega_c\left(\cos\Omega t\mathbf{e}_x+\sin\Omega t\mathbf{e}_y\right)$$



$$\begin{aligned}
\frac{q}{m}\mathbf{A}_s &= \omega_c\left(y\cos\Omega t-x\sin\Omega t\right)\mathbf{e}_z\\
\frac{q}{m}\mathbf{A}_a &= \omega_c\left(z\sin\Omega t\mathbf{e}_x-z\cos\Omega t\mathbf{e}_y\right)\\
\frac{q}{m}\mathbf{A}_c &= \frac{\omega_c}{2}\left[z\sin\Omega t\mathbf{e}_x-z\cos\Omega t\mathbf{e}_y+\left(y\cos\Omega t-x\sin\Omega t\right)\mathbf{e}_z\right]
\end{aligned}$$

Fisch, N.J. & Watanabe, T. 1982 Field reversal by rotating waves. *Nuclear Fusion* **22** (3), 423-427.

3 basics rotating fields



Electric node of a surface Alfven standing wave
Squirrel cage axial currents
Dephased orthogonal coils

$$\frac{q}{m}\mathbf{b}\left(t\right)=\omega_c\left(\cos\Omega t\mathbf{e}_x+\sin\Omega t\mathbf{e}_y\right)$$

1-Dephased orthogonal coils

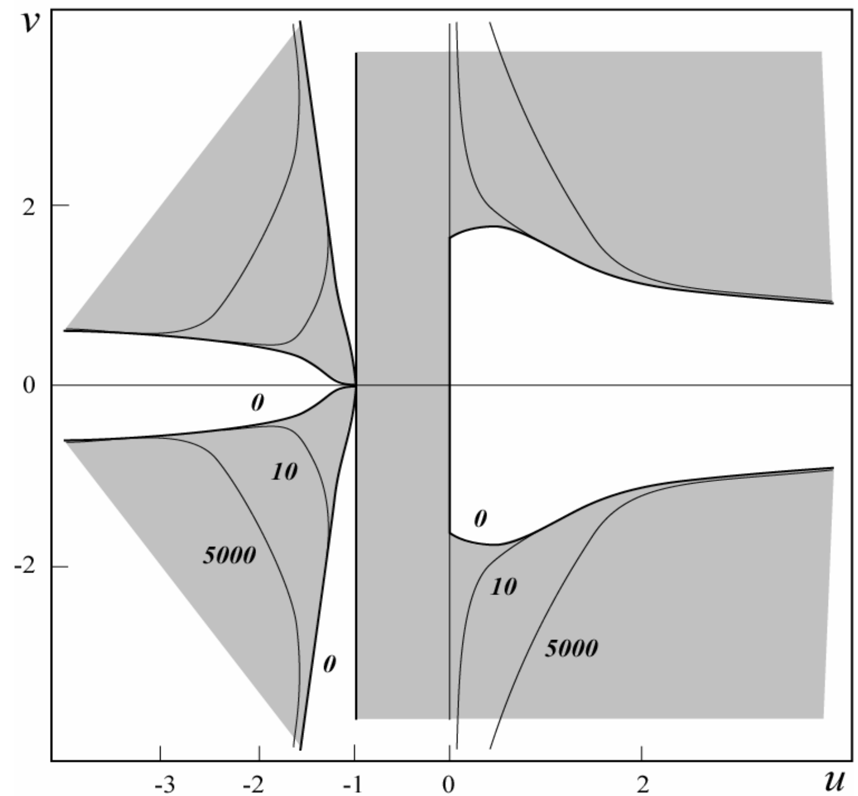
$$\boxed{\frac{q}{m}\mathbf{A}_c = \frac{\omega_c}{2}\left[z\sin\Omega t\mathbf{e}_x - z\cos\Omega t\mathbf{e}_y + (y\cos\Omega t - x\sin\Omega t)\,\mathbf{e}_z\right]}$$

$$\begin{aligned}
\frac{d^2x}{dt^2} &= \Omega_c\frac{dy}{dt} - \omega_c\frac{dz}{dt}\sin\Omega t - \frac{\Omega\omega_c}{2}z\cos\Omega t\\
\frac{d^2y}{dt^2} &= -\Omega_c\frac{dx}{dt} + \omega_c\frac{dz}{dt}\cos\Omega t - \frac{\Omega\omega_c}{2}z\sin\Omega t\\
\frac{d^2z}{dt^2} &= \omega_c\left(\frac{dx}{dt}\sin\Omega t - \frac{dy}{dt}\cos\Omega t\right) + \frac{\Omega\omega_c}{2}(y\sin\Omega t + x\cos\Omega t)
\end{aligned}$$

$\exp j\omega t$

$$\boxed{u = \frac{\Omega}{\Omega_c},\ v = \frac{\omega_c}{\Omega_c},\ w = \frac{\omega}{\Omega_c}}$$

$$w^6 - \left((1+u)^2 + u^2 + v^2\right)w^4 + u^2\left((1+u)^2 + \frac{3v^2}{4}\right)w^2 - u^3v^2\frac{1+u}{4} = 0$$



2-Electric node of a surface Alfven standing wave

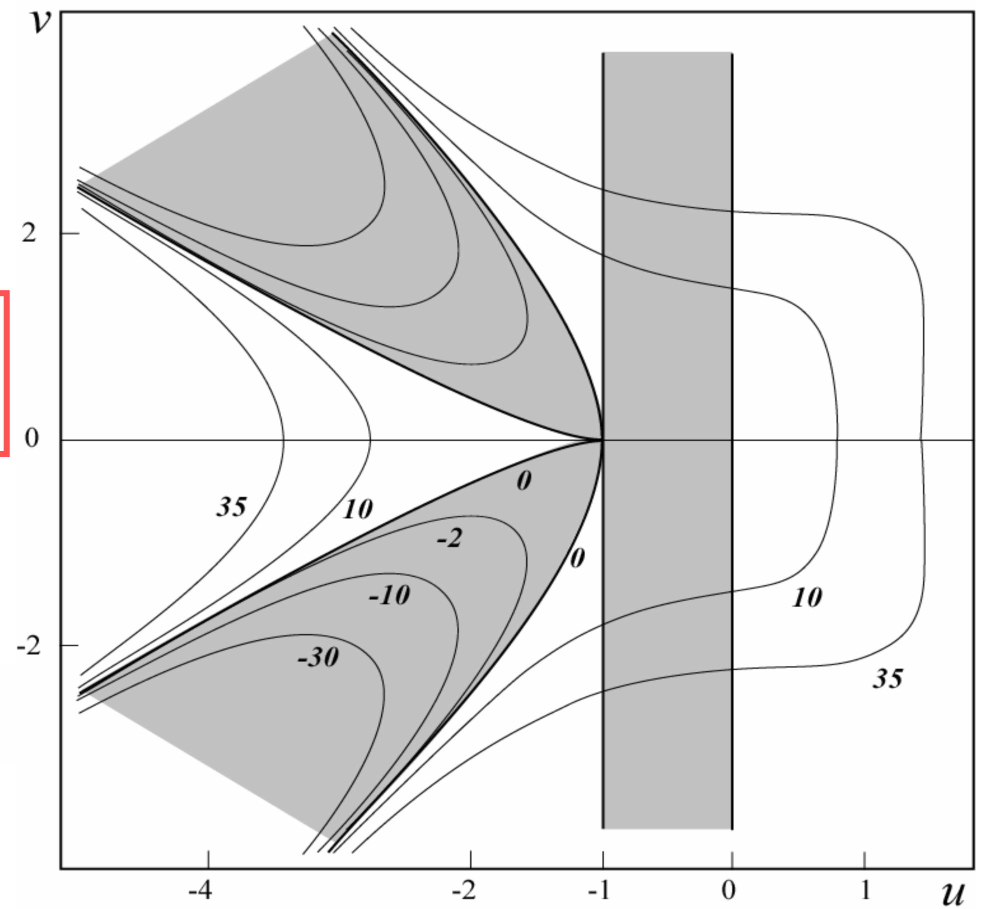
$$\boxed{\frac{q}{m}\mathbf{A}_a = \omega_c \left(z \sin \Omega t \mathbf{e}_x - z \cos \Omega t \mathbf{e}_y \right)}$$

$$\begin{aligned}
\frac{d^2x}{dt^2} &= \Omega_c \frac{dy}{dt} - \omega_c \frac{dz}{dt} \sin \Omega t - \omega_c \Omega z \cos \Omega t \\
\frac{d^2y}{dt^2} &= -\Omega_c \frac{dx}{dt} + \omega_c \frac{dz}{dt} \cos \Omega t - \omega_c \Omega z \sin \Omega t \\
\frac{d^2z}{dt^2} &= \omega_c \frac{dx}{dt} \sin \Omega t - \omega_c \frac{dy}{dt} \cos \Omega t
\end{aligned}$$

$\exp j\omega t$

$$\boxed{u = \frac{\Omega}{\Omega_c},\ v = \frac{\omega_c}{\Omega_c},\ w = \frac{\omega}{\Omega_c}}$$

$$w^4 - \left((1+u)^2 + v^2 \right) w^2 + uv^2 (1+u) = 0$$



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3-Squirrel cage axial currents

$$\boxed{\frac{q}{m}\mathbf{A}_s = \omega_c\left(y\cos\Omega t - x\sin\Omega t\right)\mathbf{e}_z}$$

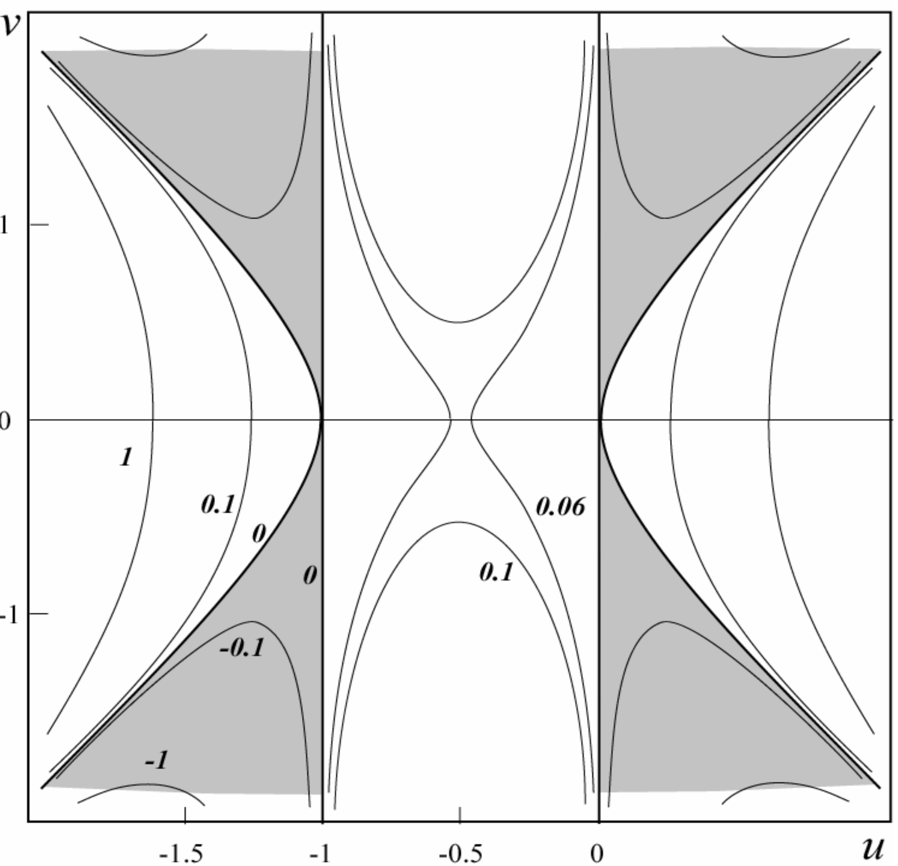
$$\frac{d^2y}{dt^2} = -\Omega_c\frac{dx}{dt} + \omega_c\frac{dz}{dt}\cos\Omega t$$

$$\frac{d^2z}{dt^2} = \omega_c\left(\frac{dx}{dt}\sin\Omega t - \frac{dy}{dt}\cos\Omega t\right) + \Omega\omega_c\left(y\sin\Omega t + x\cos\Omega t\right)$$

$$\frac{d^2x}{dt^2} = \Omega_c\frac{dy}{dt} - \omega_c\frac{dz}{dt}\sin\Omega t$$

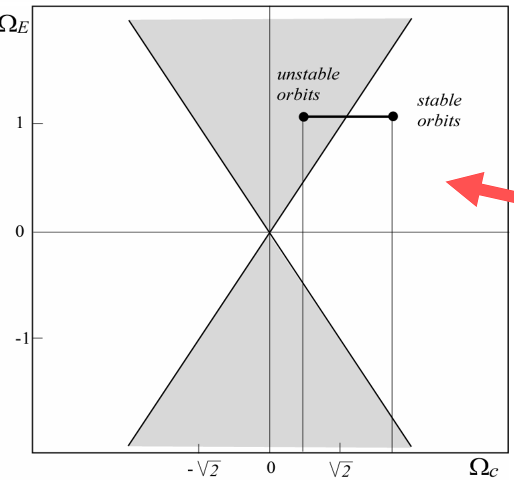
$$\downarrow \exp j\omega t$$

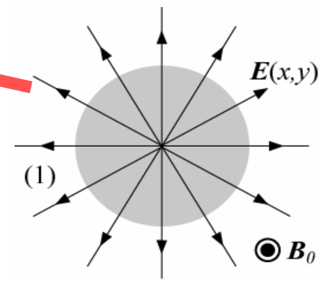
$$\boxed{u = \frac{\Omega}{\Omega_c},\ v = \frac{\omega_c}{\Omega_c},\ w = \frac{\omega}{\Omega_c}}$$

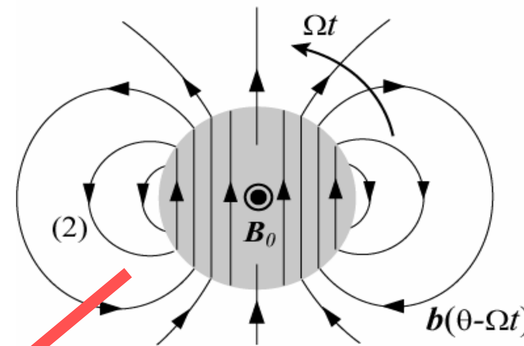


$$w^4 - \left((1+u)^2 + u^2 + v^2\right)w^2 + u(1+u)\left(u+u^2-v^2\right) = 0$$

Mass separation with rotating fields





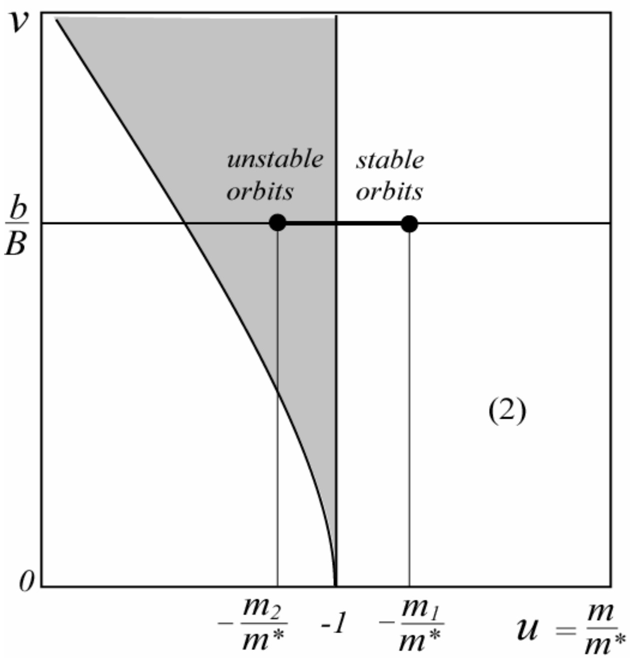


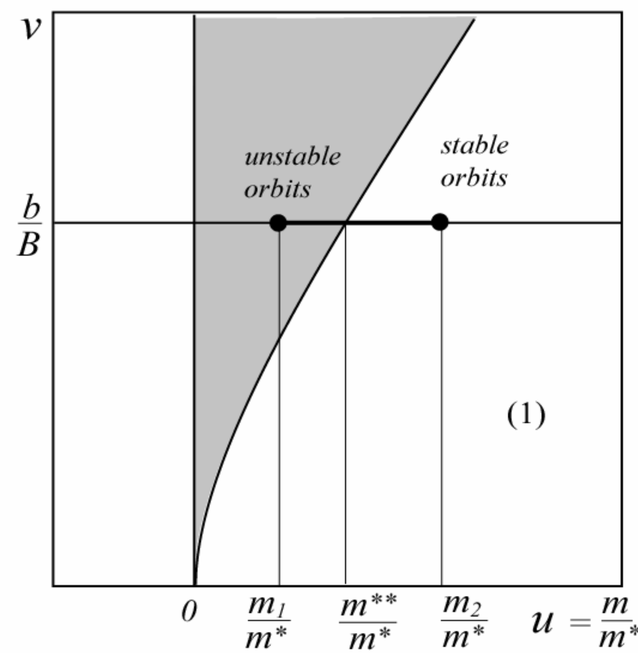
$$m^{**}(b) = m^{*}\left(\sqrt{\frac{1}{4}+\frac{b^2}{B^2}}-\frac{1}{2}\right)$$

$$m^{*} = \frac{qB_0}{\Omega}$$

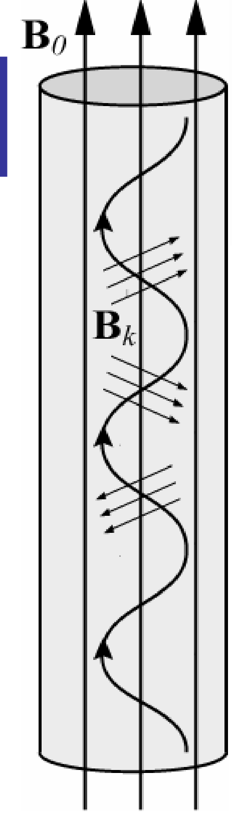
$$\boxed{v = \frac{\omega_c}{\Omega_c} = \frac{b}{B_0}}$$

$$\boxed{u = \frac{\Omega}{\Omega_c} = \frac{m}{m^{*}}}$$





$$\frac{q}{m}\mathbf{b}(t) = \omega_c\left(\cos\Omega t\,\mathbf{e}_x + \sin\Omega t\,\mathbf{e}_y\right) \longrightarrow$$ **unsolved problems**



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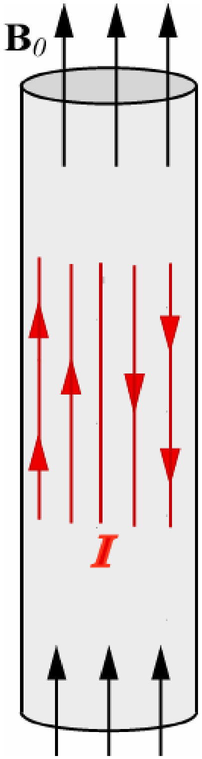
$$\frac{q}{m}\mathbf{A}_a = \omega_c\left(z\sin\Omega t\,\mathbf{e}_x - z\cos\Omega t\,\mathbf{e}_y\right)$$

$$\frac{q}{m}\mathbf{A}_a^* = \frac{\omega_c}{k_A}\left(\sin k_A z\sin\Omega t\,\mathbf{e}_x - \sin k_A z\cos\Omega t\,\mathbf{e}_y\right)$$



$$\frac{q}{m}\mathbf{A}_s = \omega_c\left(y\cos\Omega t - x\sin\Omega t\right)\mathbf{e}_z$$

$$\frac{q}{m}\mathbf{A}_s^* = \frac{\omega_c}{k_p}\left(\sinh k_p y\cos\Omega t - \sinh k_p x\sin\Omega t\right)\mathbf{e}_z$$

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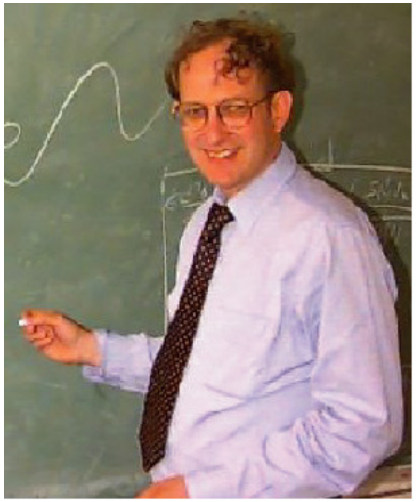
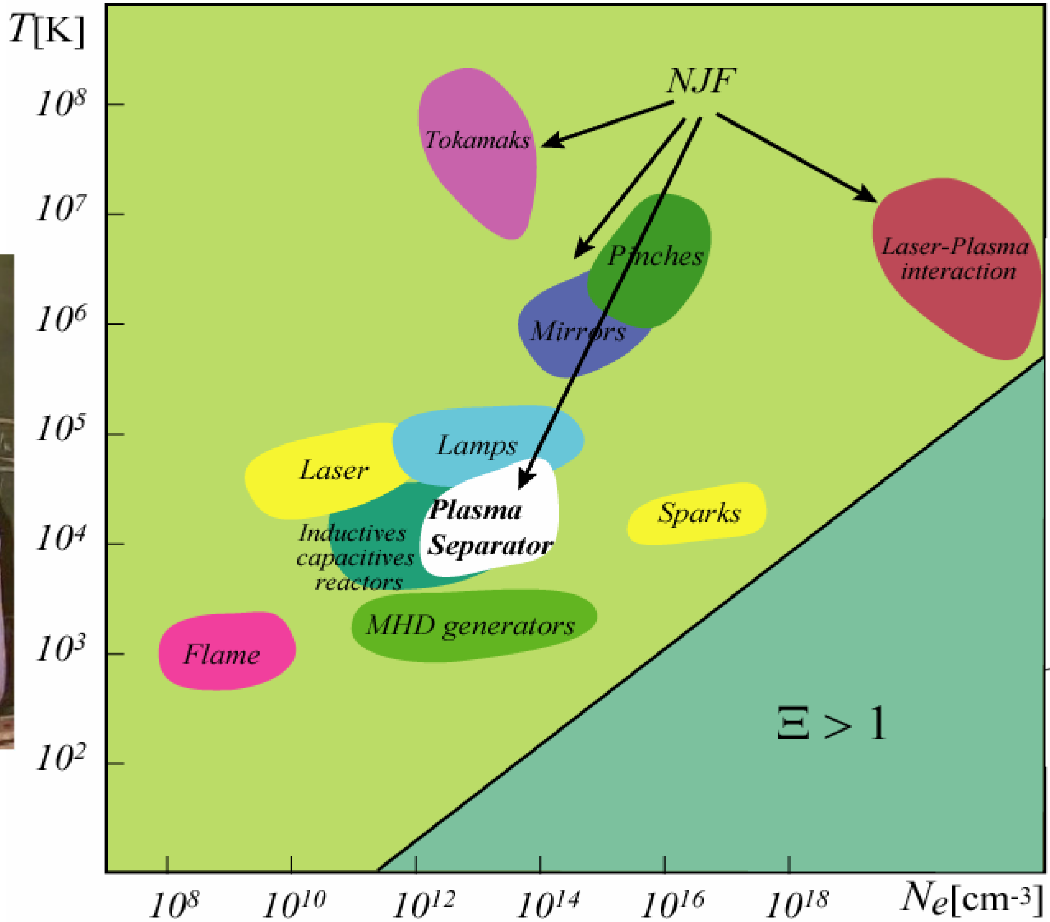
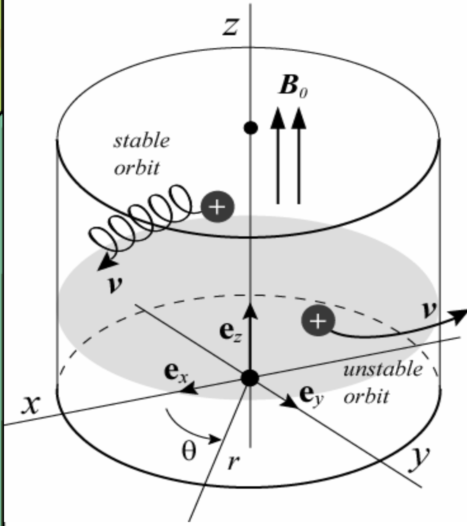
Solved or unsolved problems in plasma physics







Gueroult, R.. & Fisch, N.J. 2012 Practical considerations in realizing a magnetic centrifugal mass filter. *Phys. Plasmas* **19**, 122503.

Fetterman, A.J. & Fisch, N.J. 2011 Metrics for comparing plasma mass filters. *Phus. Plasmas* **18**, 103503.