Magnetic Field Penetration into Low Resistivity Plasma

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H.I.T. - Holon Institute of Technology

A symposium in honor of Nathaniel J. Fisch
Solved and Unsolved Problems in Plasma Physics
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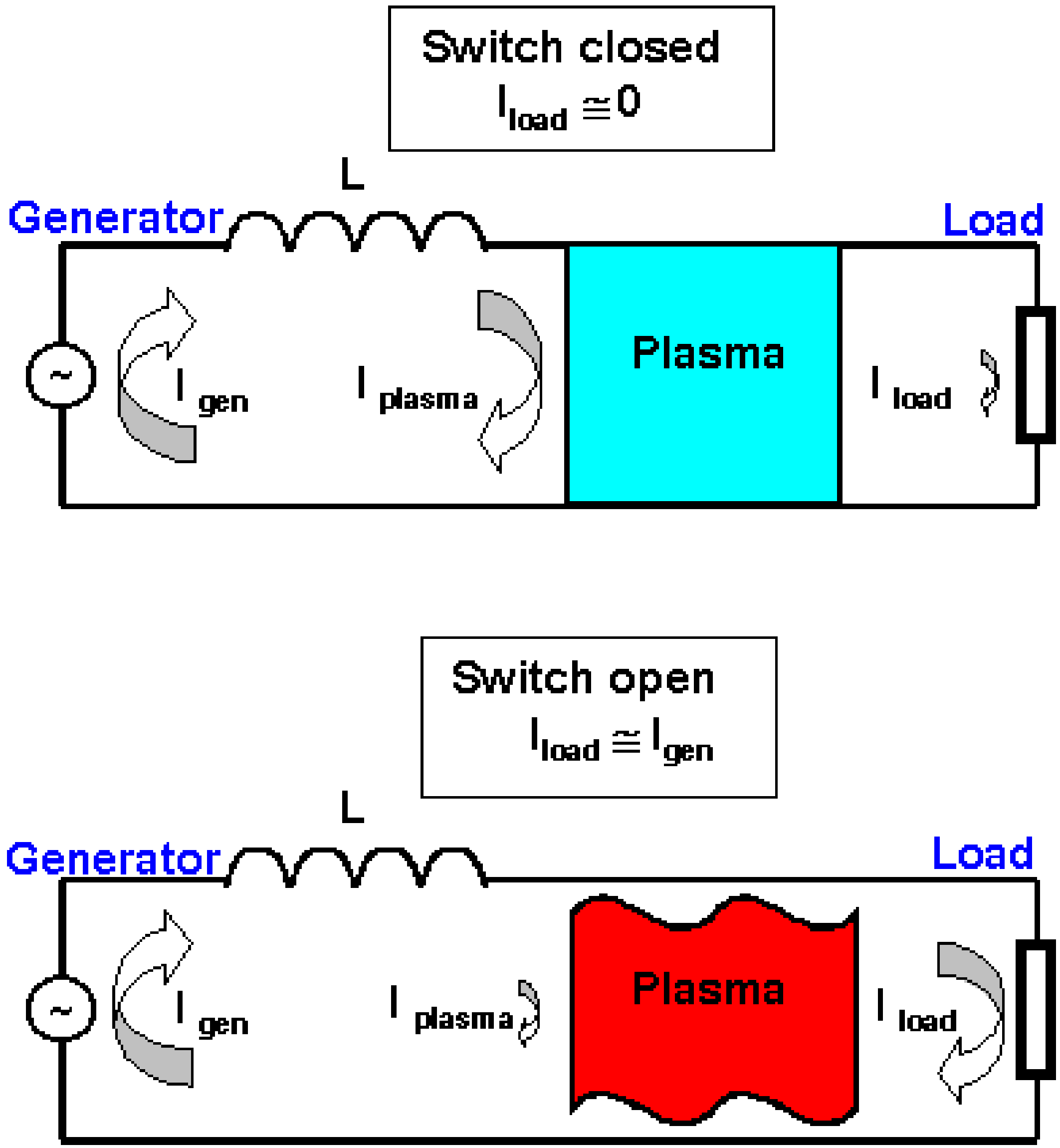
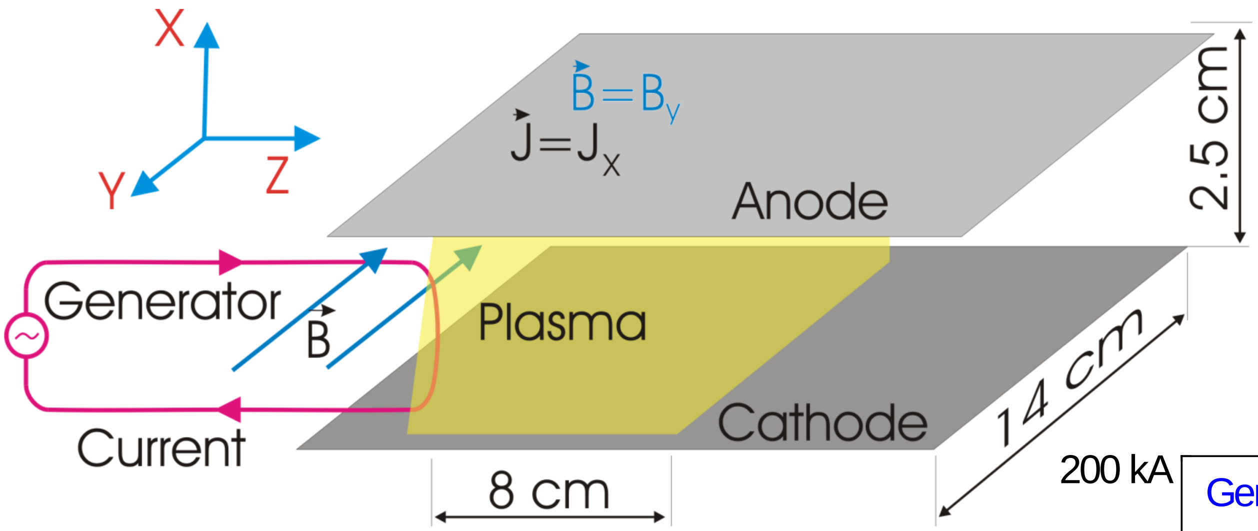
Content

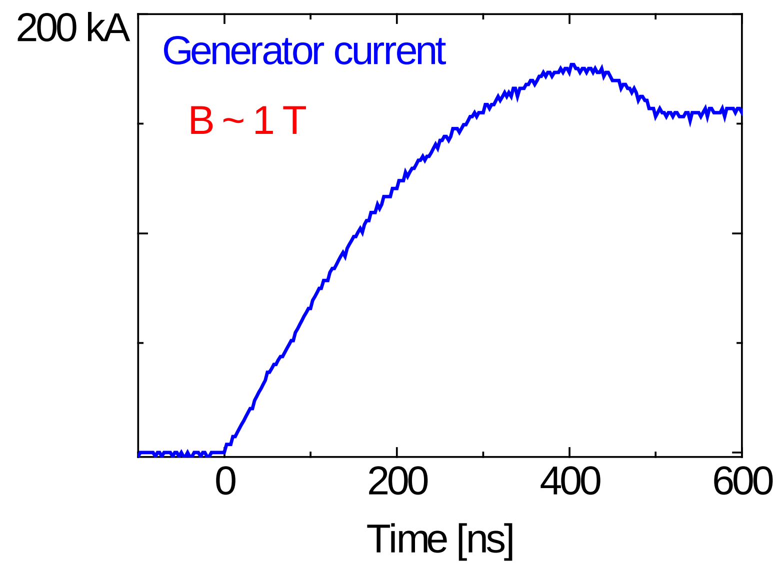
- Magnetic field penetration into plasma, much faster than expected by resistive diffusion, has been measured.
- It is explained as a Hall-induced penetration in a non-uniform plasma. A shock is generated with a large deviation from the frozen-in law and a large dissipation.
- Newly discovered ion-species separation, and yet unsolved problems.

Basic configuration - Plasma Opening Switch (POS)



Experiment – Plasma Opening Switch





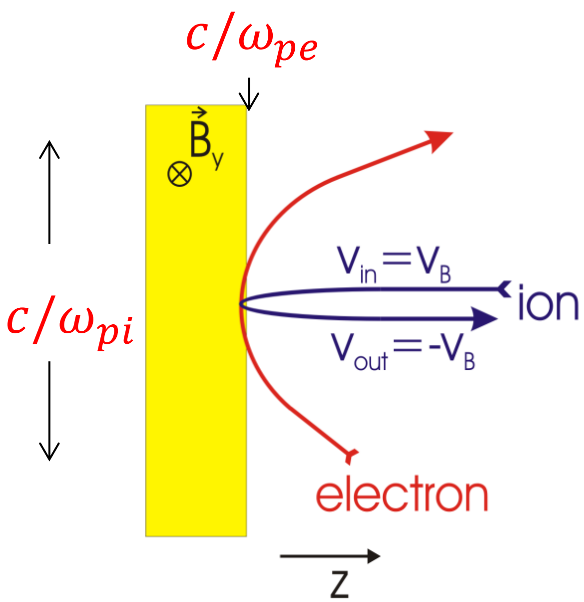
Often cylindrical geometry

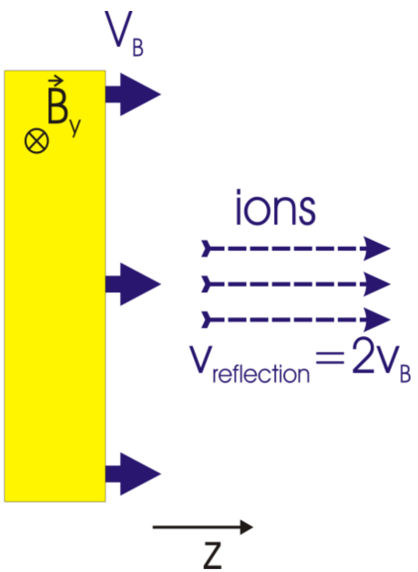
Magnetic field – plasma interaction

Two competing processes

- Plasma pushing by magnetic pressure
- Magnetic field penetration

Collisionless plasma - pushing by magnetic pressure





$$v_B = \frac{B}{\sqrt{2\mu_0 m_i n}}$$

In the frame
of moving B-field

Laboratory
frame

Specular reflection in the moving frame
(Like a moving wall)
Dissipated magnetic field energy =
ion directed kinetic energy

Rosenbluth (1954)

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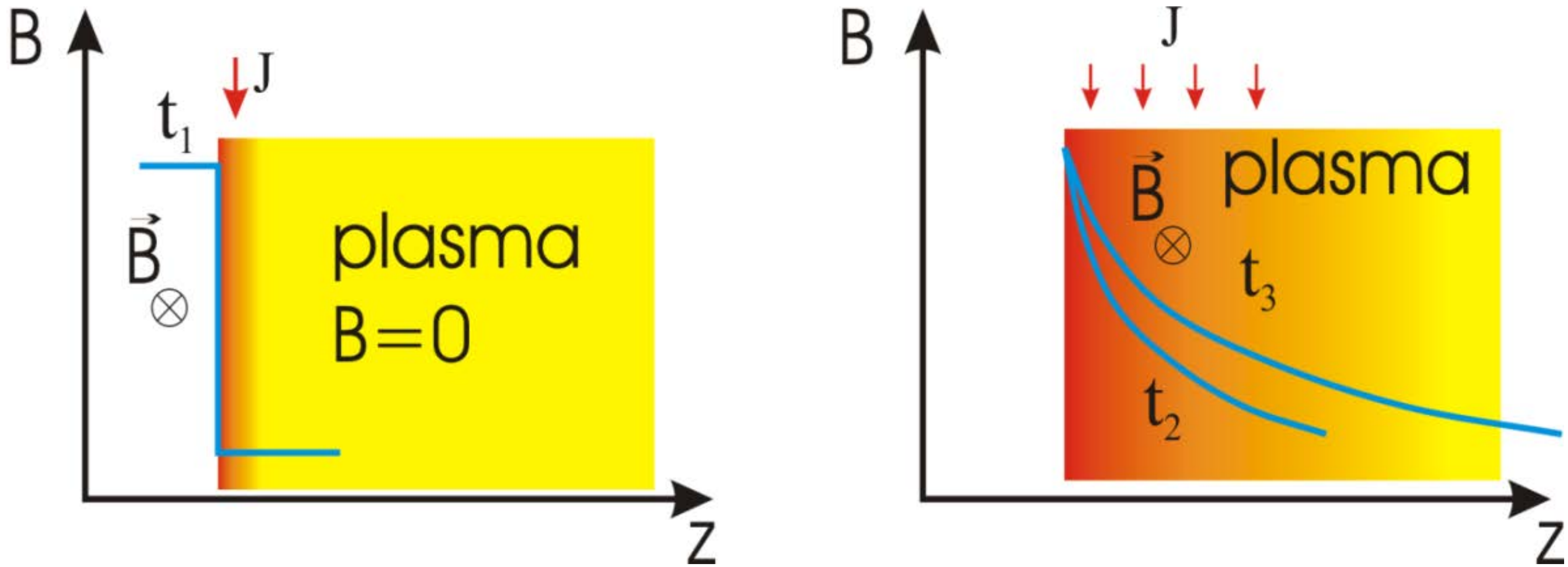
Highly-resistive plasma → Magnetic field penetration by diffusion



Dissipated magnetic field energy = Joule heating

$$v_D = \frac{\eta}{\mu_0 l}$$

Dominant process

- Plasma pushing by magnetic pressure dominant when $$v_B = \frac{B}{\sqrt{2\mu_0 m_i n}} \gg v_D = \frac{\eta}{\mu_0 l}$$
- Magnetic field penetration

$$v_D = \frac{\eta}{\mu_0 l} \gg v_B = \frac{B}{\sqrt{2\mu_0 m_i n}}$$

The Puzzle

Low-resistivity plasma

Fast magnetic field penetration measured
Spectroscopy – Maron's group, Weizmann (1993)

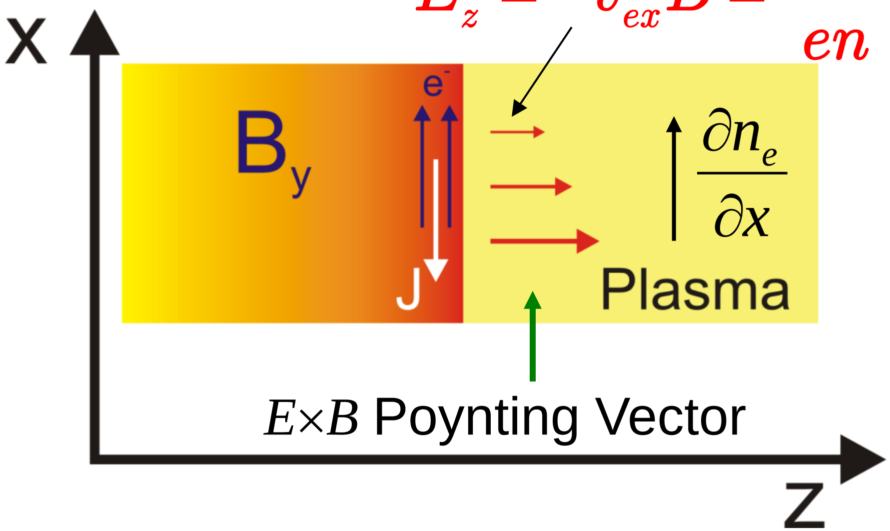
The explanation:

Penetration due to the Hall-Field mechanism

Plasma nonuniformity (and finite resistivity)

2D effect

$$E_z = -v_{ex} B = \frac{j_x B}{en}$$



The charge-separation axial field becomes inductive due to density gradient in the x direction. A loop voltage is formed.

Gordeev, Rudakov, Kingsep

Fruchtman Gomberoff

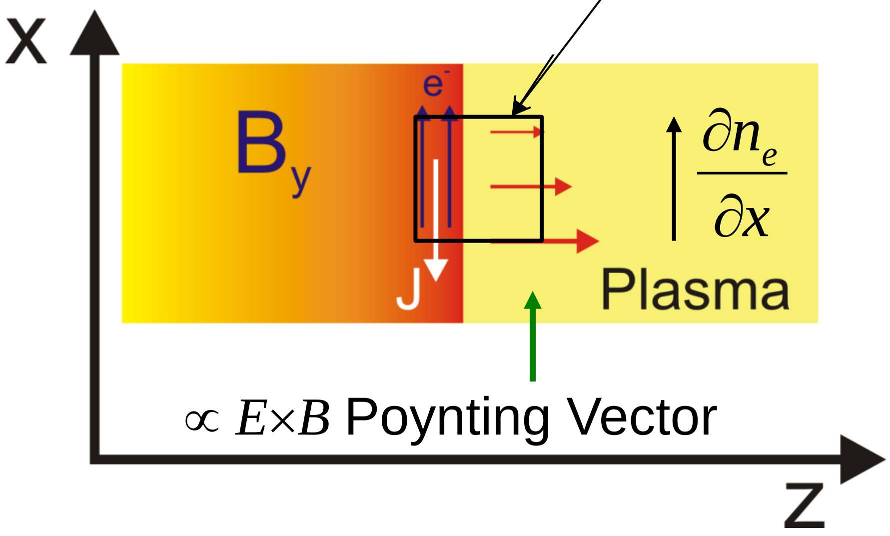
1980's1990'

Penetration due to the Hall-Field mechanism

Plasma nonuniformity (and finite resistivity)

$$\Delta\varphi = \frac{B_0^2}{2\mu_0 e n}$$

2D effect



The charge-separation axial field becomes inductive due to density gradient in the x direction.
A loop voltage is formed.

Hall-field magnetic field penetration

- A quasi-1D penetration axially – z direction- while the energy flows in the x direction.
- A traveling wave solution exists, a solution of Burgers equation:

$$\frac{\partial B}{\partial t}+\left[-\frac{1}{e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\right]B\frac{\partial B}{\partial z}=\frac{\eta}{\mu_0}\frac{\partial^2 B}{\partial z^2}$$

$$B\rightarrow B_0 \quad z\rightarrow -\infty$$

$$B\rightarrow 0 \quad z\rightarrow \infty$$

A traveling wave solution:

$$v_H \equiv -\frac{B_0}{2e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)$$

Hall-field magnetic field penetration

$$\frac{\partial B}{\partial t}+\left[-\frac{1}{e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\right]B\frac{\partial B}{\partial z}=\frac{\eta}{\mu_0}\frac{\partial^2 B}{\partial z^2}$$

$$B\to B_0 \quad z\to-\infty \qquad B\to 0 \quad z\to\infty$$

A traveling wave solution:

$$B=\frac{B_0}{2}\left\{1-\tanh\left[\frac{1}{d}\left(z-v_H t\right)\right]\right\}$$

$$v_H\equiv-\frac{B_0}{2e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\cong\frac{B_0}{2en\mu_0 l_x}$$

$$d\equiv\frac{4en\eta}{B_0}l_x$$

Frozen-in law

- Stimulating questions about the frozen-in law were raised by Nat Fisch when this research began.

Frozen-in law for collisionless plasma

The magnetic field increase =

a combination of flux compression and

flux penetration.

In this configuration:

$\frac{\boldsymbol{B}}{\boldsymbol{n}} = \boldsymbol{const}$. Along electron trajectory.

Magnetic field increases in time as electron moves from a low density region to a high density region. magnetic flux compression.

However, the magnetic field is initially zero. There must be magnetic flux penetration.

The frozen-in law

magnetic flux penetration

$$\left[\frac{B}{n}\right] = \frac{2\mu_0}{B_0} \int \frac{\eta j^2}{n} dt$$

Associated with energy dissipation.

more puzzles

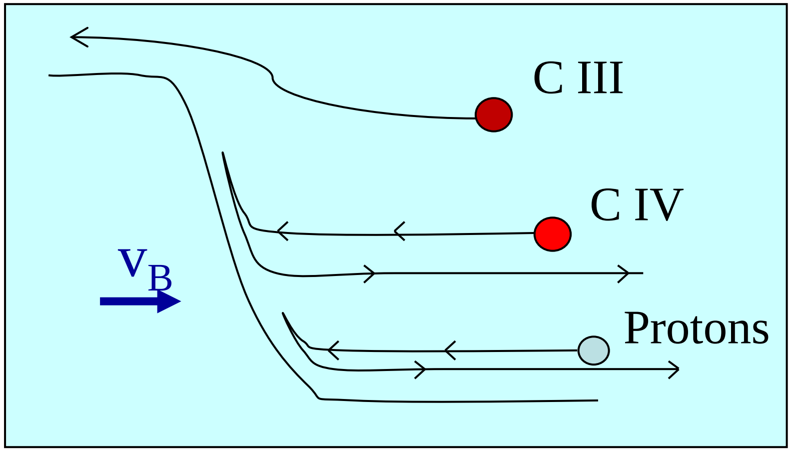
- The Hall effect mechanism relies on density nonuniformity in a specific direction.
- Thorough spectroscopic measurements at Weizmann showed magnetic field penetration also in cases in which density nonuniformity is small.
- Ion separation was observed.

Electric potential hill and simulated density

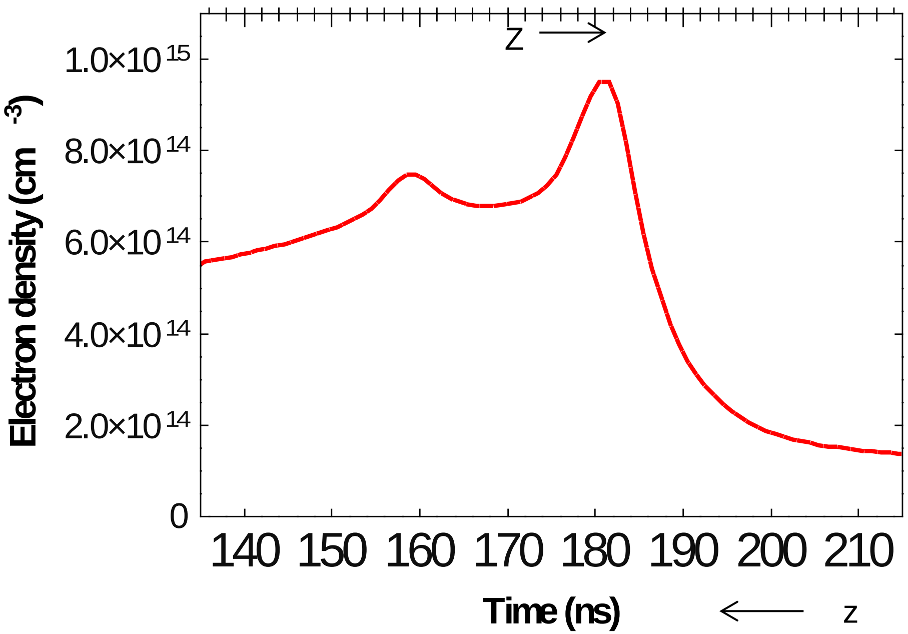
The ion velocities propagate as a front that travels at a constant velocity imply the presence of an electric potential hill ϕ (in the rest frame of the propagating field).

$$Z_i e\phi(t) = \tfrac{1}{2} m_i \mathrm{v}_{B-field}^2 - \tfrac{1}{2} m_i \left(\mathrm{v}_i(t) - \mathrm{v}_{B-field}\right)^2$$

By setting $V_i = V_{\text{B-field}}$ the **reflection point** of an ion *i* is determined.



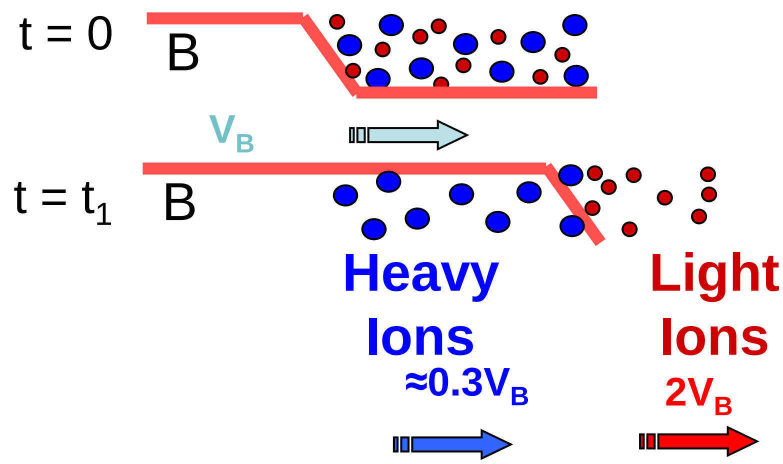
The calculated ion density gives the **electron density**, Which was confirmed independently.



MFP and ion separation (found at Weizmann)

Simultaneous rapid B-field **penetration** into the heavier-ion component and light ion **reflection**

(Weingarten et. *al*., *PRL* , 2001)



Magnetic field front and a potential hill move inside the plasma causing ion separation (Rubinstein et. al. ,2016)

Ion separation and magnetic field penetration

- We understand ion separation once magnetic field penetration occurs, but what causes the magnetic field penetration (if the density is initially uniform)?

The suggested explanation for ion separation and magnetic field penetration

- The electron density is modified by the pushing by the MF.
- If the plasma is composed of , say, two ion species, and if the composition is nonuniform, then the different pushing of the different ion species by the magnetic field may generate the electron density nonuniformity that allows Hall-field MFP.

(AF, Doron, Maron, 2014)

Two cases

- Case 1:

The density of the light ions increases in the direction of the electron flow (while the density of the heavy ions decreases).

- Case 2:

The density of the heavy ions increases in the direction of the electron flow (while the density of the light ions decreases).

case 1

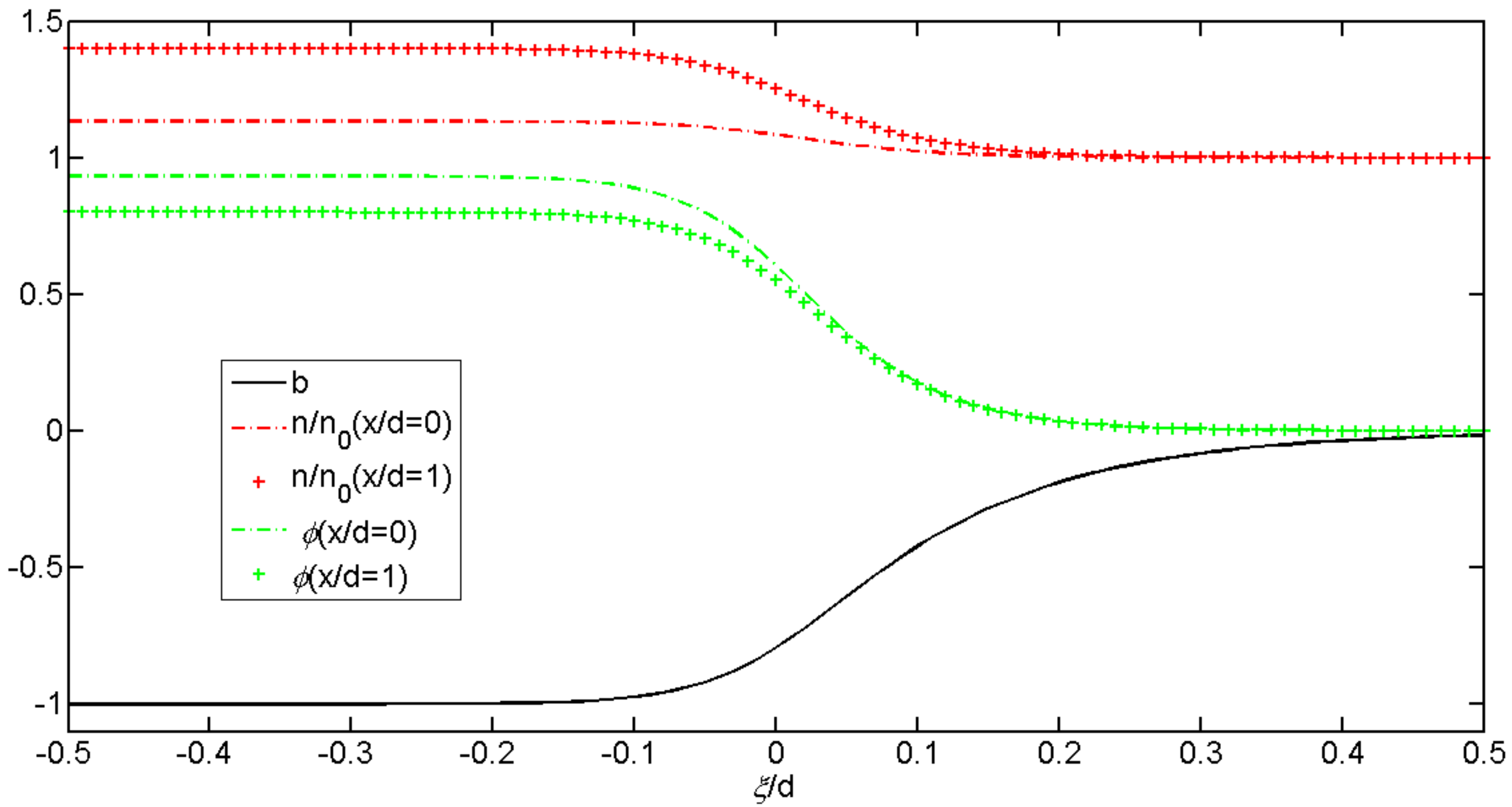
The density of the light ions increases in the direction of the electron flow (while the density of the heavy ions decreases).

The light ions slow down more than the heavy ions due to the potential hill and their density increases more than the density of the heavy ions increases. Therefore the total density where the light ions density dominate increases. A density nonuniformity is generated that allows Hall MFP.

In the moving frame

- If we look at the distributions in the frame moving with the wave, the solution is stationary. The electric field is static and is described by a potential. There is no loop voltage because of the motional electric field.

Case 1 – The B field moves to the right, In the wave frame, it is stationary. All ions move to the left and climb the potential hill. The density near the anode becomes higher since the light ions slow down more than the heavy ions. The potential becomes higher near the cathode



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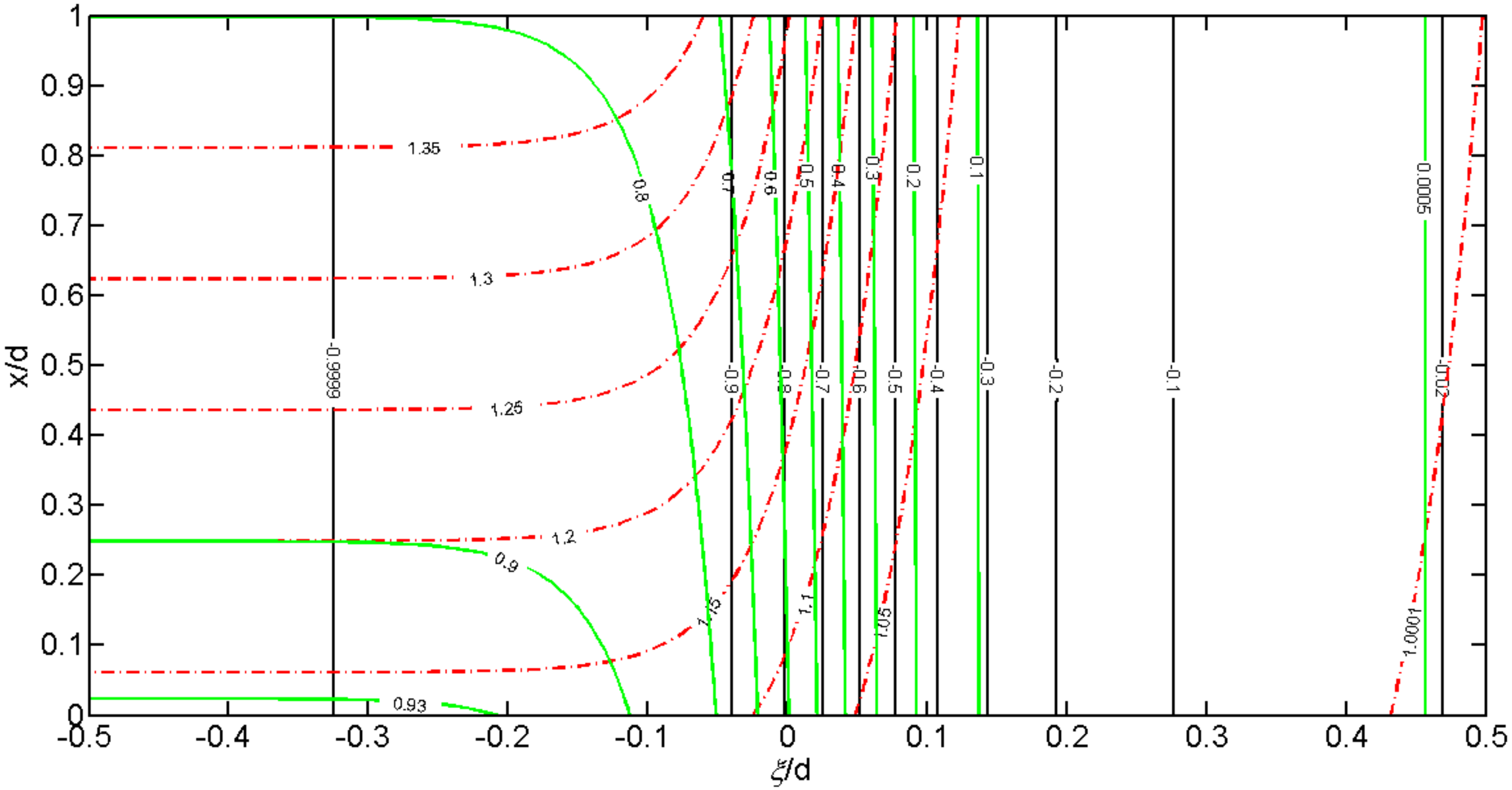
Case 1 – density initially uniform. Density gradient upwards (towards anode), moving frame - no inductive electric field

density potential



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Case 1 – analytical description

$$\frac{\partial b}{\partial t} = \left(\frac{v_{Al}^2 v_{H1}}{v_B^2} \right) \frac{\partial b^4}{\partial z} + \frac{\eta}{\mu_0} \frac{\partial^2 b}{\partial z^2}$$

v_{H1} **velocity due to density gradient**

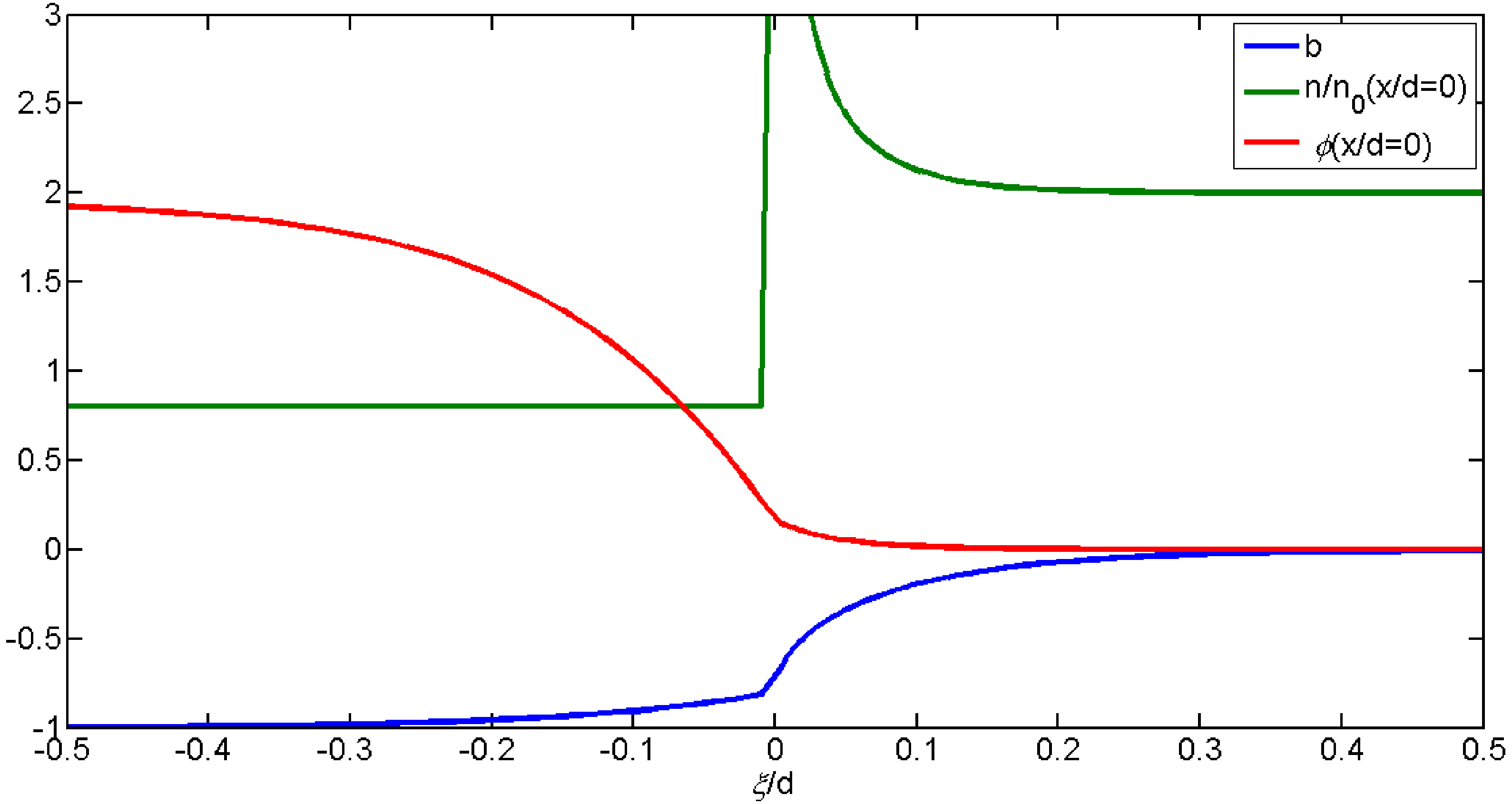
v_{Al} **hydrodynamic velocity** $\quad b \equiv \frac{B}{B_0} \quad \xi \equiv z - v_B t$

$$\Rightarrow b = -\frac{1}{\left[1 + \exp\left(3\mu_0 v_B \xi / \eta\right)\right]^{1/3}}, \quad v_B = \left(v_{Al}^2 v_{H1}\right)^{1/3}$$

Case 2

- The density of the heavy ions increases in the direction of the electron flow (while the density of the light ions decreases).
- The light ions are reflected by the magnetic field.
- The magnetic field penetrates the heavy ions, because of their density gradient.

Case 2



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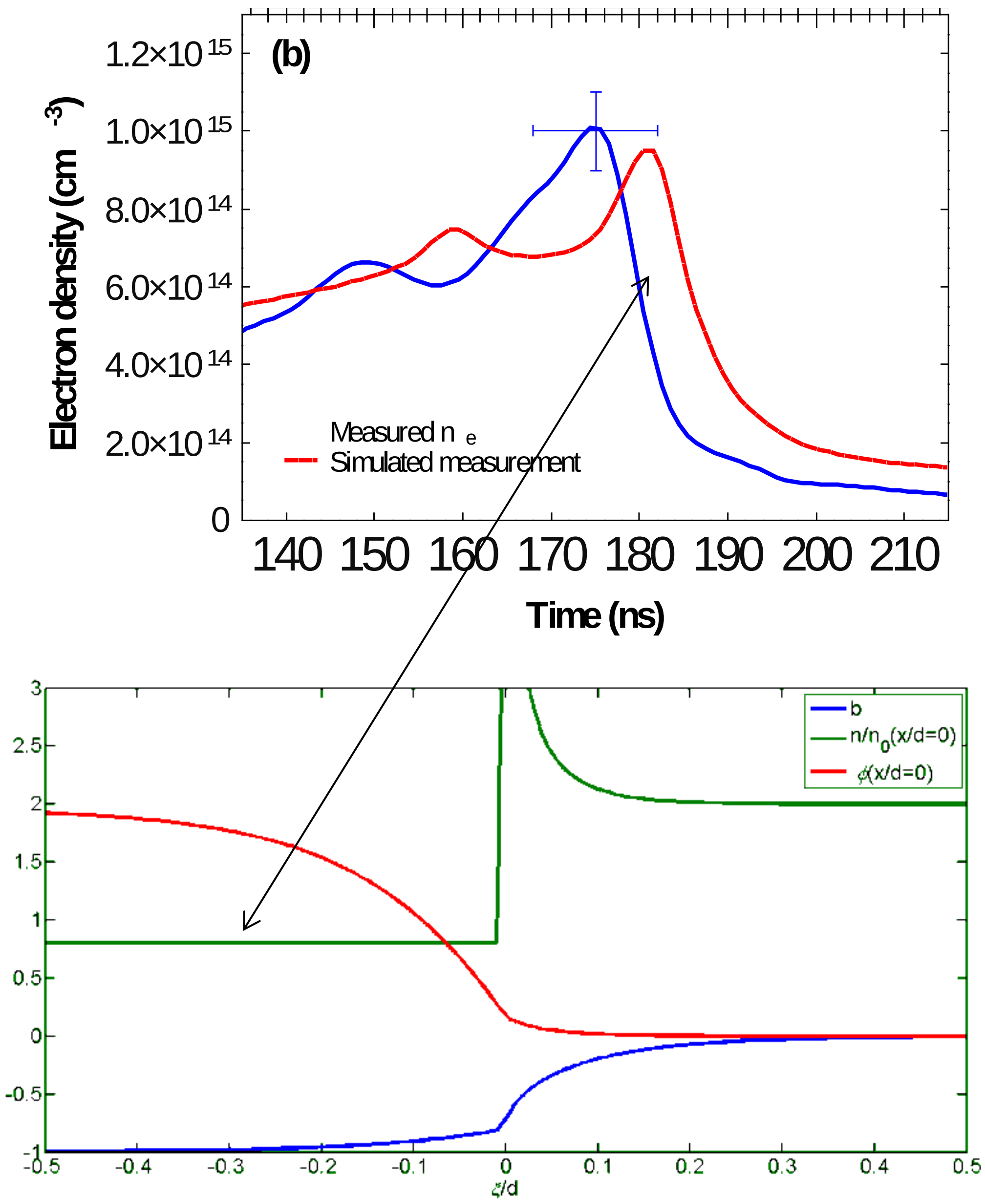
Density profiles – experiment and theory



B moves ←

B moves →

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Unsolved problem -1

The fate of the dissipated magnetic field energy: The magnetic field penetration should result in a large dissipation of magnetic field energy. In the shock front a large Joule heating is expected, even if the resistivity is small. Electrons do not seem to heat to the keV level, that is expected by energy conservation.

Unsolved problem - 2

Symmetrical penetration: Theory predicts that for certain density gradients, there should not be any magnetic field penetration. Although we showed that a favorable density gradient may evolve due to plasma pushing for some cases, it is desirable to understand why such a magnetic field penetration seems to always occur.

Thank you

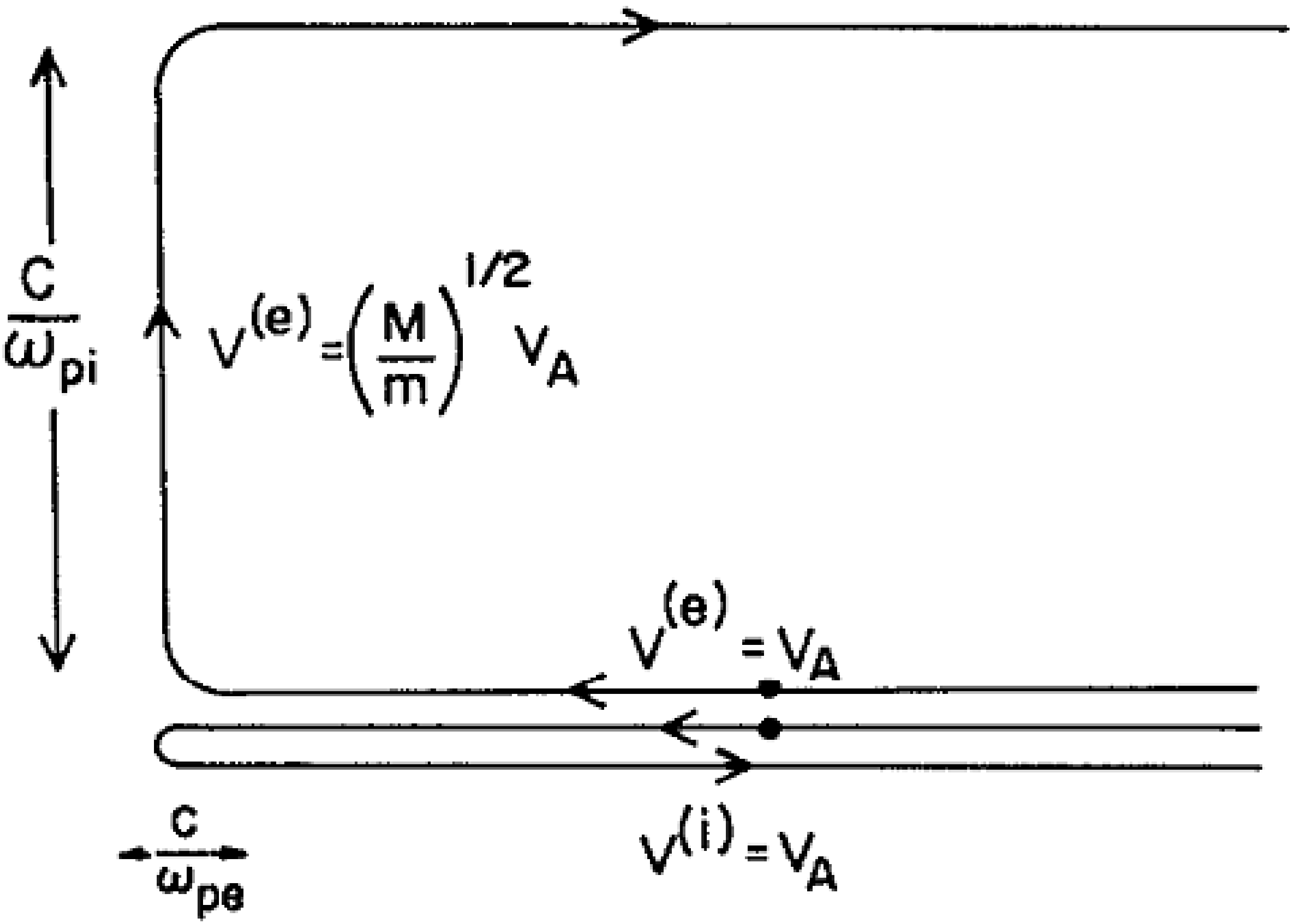
and

best wishes to Nat Fisch!

Case 2 – region 1

- The light ions are reflected by the magnetic field.
- We assume that the density of the reflected light ions is much larger than that of the heavy ions. We neglect the heavy ions in this region.
- We adopt the Rosenbluth analysis (1954) of the reflected ions.

Plasma specular reflection



Case 2 – region 1 – specular reflection

$$\frac{\partial^2 A}{\partial \xi^2} = \frac{2\omega_p^2}{c^2\sqrt{1-\left(A/A_r\right)^2}}A$$

$$\ln\tan\left[\frac{1}{4}\arcsin\left(\frac{A}{A_r}\right)\right] + 2\cos\left(\frac{1}{2}\arcsin\left(\frac{A}{A_r}\right)\right) - 0.53284 = -\frac{\xi\sqrt{2}\omega_p}{c}$$

$$B = -B_r\sqrt{1-\sqrt{1-\left(\frac{A}{A_r}\right)^2}}\;;\; n = \frac{2n_0}{\left[1-\left(B/B_r\right)^2\right]};\quad \phi = \phi_r\left\{1-\left[1-\left(\frac{B}{B_r}\right)^2\right]\right\}^2,$$

$$A_r \equiv \left(v_B/e\right)\sqrt{m_e m_l/Z_l}$$

$$B_r \equiv 2\left(\omega_p/c\right)A_r = 2v_B\sqrt{n_0 m_l \mu_0/Z_l}$$

$$\phi_r \equiv m_l v_B^2/2Z_l e$$

Case 2 – region 2 – heavy ions

$$\frac{\partial b}{\partial t} = v_{H2}\frac{\partial b}{\partial z}b + \frac{\eta}{\mu_0}\frac{\partial^2 b}{\partial z^2}, \qquad v_{H2} \equiv -\frac{B_0}{2Z_h e\mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n_{h0}}\right) \geq 0$$

Continuity of b and E_x

$$b = \frac{u^2(1-u) - u(1+u)\exp\left[\left(-u^2-1\right)\varsigma\right]}{(1-u) + u(1+u)\exp\left[\left(-u^2-1\right)\varsigma\right]}$$

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Case 2 - Matching the 2 regions

The normalized velocity:

$$u \equiv \frac{v_B}{v_{Al}} = -\frac{v_{Al}}{2v_{H2}} + \sqrt{\left(\frac{v_{Al}}{2v_{H2}}\right)^2 + 1}, \qquad \varsigma \equiv \frac{v_{H2}\mu_0}{\eta}\xi$$

$$v_{Al} \equiv \left(\frac{B_0}{2}\right)\sqrt{\frac{1}{n_l m_l \mu_0}}$$

v_{H2} v_{Al} **determine** v_B

$$\varphi = \left(\frac{u}{2}\right)^2\left[1 - \left(1 - \frac{b^2}{u^2}\right)^2\right] \quad \zeta \geq 0, \qquad \varphi = \left(\frac{u}{2}\right)^2 + p_2(x)\left(b^2 - u^2\right) \quad \zeta \leq 0$$

$$p_2(x) \equiv Z_l n_{l0} / Z_h n_{h0}(x)$$

Case 2 – matching the two regions

The normalized velocity:

$$u \equiv \frac{v_B}{v_{Al}} = -\frac{v_{Al}}{2v_{H2}} + \sqrt{\left(\frac{v_{Al}}{2v_{H2}}\right)^2 + 1},$$

$$v_{Al} \equiv \left(\frac{B_0}{2}\right)\sqrt{\frac{1}{n_l m_l \mu_0}}$$

$$v_{H2} \equiv -\frac{B_0}{2Z_h e \mu_0}\frac{\partial}{\partial x}\left(\frac{1}{n_{h0}}\right) \geq 0$$

v_{H2} v_{Al} **determine** v_B

$$P_l = B_r^2 / B_0^2 = u^2$$

$$\varphi_r = (u/2)^2$$

$$\varphi_{\max} = (u/2)^2 + p_2(1 - u^2)$$

$$p_2(x) \equiv Z_l n_{l0} / Z_h n_{h0}(x)$$