Current drive for stabilizing magnetic islands: topological and geometrical aspects

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The problem of island control

- Classical and neoclassical tearing modes are a serious cause of degradation of plasma confinement and an important open issue is to find the appropriate means of controlling them.

- One of the most promising methods is based on the injection of an external control current within the magnetic island (idea based on the seminal paper by Fisch PRL 78).

- Due to its localized deposition, the electron cyclotron current drive (ECCD) is very appropriate for this stabilization purpose, since the driven current can both modify locally the equilibrium and counteract the unstable perturbation evolving into a magnetic island.

- Experimental results have demonstrated successful stabilization on several devices (ASDEX, DIII-D, FTU, JT-60, TCV, TEXTOR see review by La Haye PoP06).
The problem of island control

- Many open questions on:
  1. estimate of the required current drive
  2. effects of the radio frequency beam profile
  3. drift tearing effects
  4. nonlinear effects such as shielding currents sheets and plasmoid formation
  5. effect of anisotropic transport
  6. effect of flip (phase) instabilities
  7. tempertaure effects
  8. ...........

- For all these reasons the final goal of realizing a robust real time control is still unattained!
The problem of island control

- Conventional approaches are based on the generalized Rutherford equation which describes the time evolution of the island width \( w \)

\[
\frac{g_1^1}{\eta} \frac{dw}{dt} = \Delta_{eq}' + \Delta_{bs}' + \Delta_p' + \Delta_{cd}' + \ldots
\]

- Where: \( w \) is the amplitude island width depending on the magnetic perturbation

\( g_1 \) = constant

\( \Delta_{eq}' \) = standard tearing instability parameter of the unperturbed equilibrium

\( \Delta_{bs,p,cd}' \) = modifications of the instability parameter due the bootstrap, polarization, current driven currents
The problem of island control (a small contribution...)

- The island concept itself is intrinsically two-dimensional.
- The approach based on the generalized Rutherford equations neglects the 2D topological aspects of the problem.
- We consider here the most simple situation and configuration that allow to take into accounts these aspects:
  
  - RMHD equations in slab configuration
  - Unstable equilibrium
  - Tearing modes which give raise to magnetic island
  - Injection of a current simulating the ECCD in the island

- This is a basis for the understanding of more complex models.
The model

- RMHD:

\[ \frac{\partial \psi}{\partial t} + \mathbf{v}_\perp \cdot \nabla \psi = -\eta (J - J^{(0)} - J_{ec}) \]

\[ \frac{\partial U}{\partial t} + \mathbf{v}_\perp \cdot \nabla U = \mathbf{B}_\perp \cdot \nabla J. \]

\[ \mathbf{B} = B_0 \mathbf{e}_z + \mathbf{B}_\perp \quad \mathbf{B}_\perp = \nabla \psi \times \mathbf{e}_z \quad \mathbf{v}_\perp = -\nabla \phi \times \mathbf{e}_z \]

\[ U = \nabla_\perp^2 \phi \quad J = -\nabla_\perp^2 \psi \]

- The aim of the control current is to restore the ideal frozen flux condition in Eq.(1), by reducing the perturbed current density \( J - J^{(0)} \). This is expected to counterbalance the unstable reconnection process and eventually to stop the magnetic island growth.
The model

- The control current density profile is assumed to be Gaussian and distributed uniformly on the constant magnetic surfaces:

$$ J_{ec}(x, y, t) = J_m(t) \exp \left( -\frac{(\psi(x, y, t) - \psi_o(t))^2}{\delta^2} \right) $$

- The peak amplitude $J_m$ has a step function time waveform

$$ J_m(t) = \begin{cases} 
  0 & t < t_1 \lor t > t_2 \\
  A & t_1 < t < t_2,
\end{cases} $$

- where $t_1$ and $t_2$ are the switching on and switching off time of the current control, respectively.

$$ A = -a \cdot (J_X(t_1) - J_0(t_1)) \quad \delta = b \cdot (\psi_X(t_1) - \psi_o(t_1)) $$

$a$ \quad amplitude \quad $b$ \quad beam width
Evolution of free system

- We set up a numerical experiment of spontaneous magnetic reconnection process in a static, Harris pinch equilibrium configuration with

\[ B_{\parallel y}^{(0)} = B_y^{(0)} \tanh(x/L) \quad \text{and} \quad v_{\perp}^{(0)} = 0 \]

where \( B_{y}^{(0)} = 1 \) and \( L = 1 \).

I – exponential growth (linear phase)

II – algebraic phase

III – exponential growth

IV – saturation regime
ECCD effects in the linear phase

- Three cases with fixed amplitude, $a=10$, and different beam widths, $b$.

Flip instability after the first island suppression

- Evaluation of the evolution equation for $\psi$ at the X- and O- points in the small island width approximation $w^2 \propto (\psi_X - \psi_O)$ gives:

$$\frac{dw^2}{dt} = 8 \eta (J_X - J_O - J_{ecX} + J_{ecO})$$
ECCD effects in the linear phase

\[ \frac{d\omega^2}{dt} = 8\eta (J_X - J_O - J_{ecX} + J_{ecO}) \]

\[ \frac{d\omega^2}{dt} < 0 \]

\[ J_X - J_O < J_{ecX} - J_{ecO} \]
ECCD effects in the linear phase

\[
\frac{dw^2}{dt} = 8 \eta \left( J_X - J_O + J_{ecX} - J_{ecO} \right)
\]

\[ J^X - J^O \]

\[ J_{ecO} - J_{ecX} \]

\[ J^X - J^O < 0 \]

\[ J_{ecX} - J_{ecO} = 0 \]

\[ w \approx 0 \quad \text{Flip occurs} \]
ECCD effects in the linear phase

\[ \frac{d\omega^2}{dt} = 8 \eta \left( J_x - J_O - J_{ecX} + J_{ecO} \right) \]

\[ \frac{d\omega^2}{dt} > 0 \]

\[ J_x - J_O > J_{ecX} - J_{ecO} \]
ECCD effects in the linear phase

- In the case $b=0.1$, corresponding to a very narrow injected current beam, new unstable modes are driven and lead to a strong modification of the magnetic island and to a total loss of the control.

- The effect of the early control action meant to restore stability may lead, on the contrary, to another unstable state.
ECCD effects in the nonlinear phase

- Here the control current is turned on when the magnetic island is large ($w \sim 1.6$).

- Four cases with fixed beam width, $b = 0.5$, and different amplitude, $a$.

- Low values of $a$ have proved rather ineffective in counteracting the main $m = 1$ component of the magnetic flux function perturbation.

- A faster island contraction is observed when $a > 3$.

- However, the nonlinear growth of higher order harmonics prevents the island suppression.
ECCD effects in the nonlinear phase

- The island nonlinear response to the ECCD injection leads to thin, bar shaped, velocity layers distributed along the $x = x_{KH}$ axes, where the higher order harmonics drive the KH.

- Secondary island chains form, that correspond to a wide spectrum of modes for the magnetic flux, with multiple $X$- and $O$- points.

- At saturation a new equilibrium is reached with a macroscopic deformation of the island.

- Such complex topology interferes with the ECCD control action.
Conclusions and future perspectives

- The ECCD control models of tearing modes based on the conventional 0-D Rutherford equation present very stringent requirements on the focusing of the EC wave beam on the magnetic island.
- The present study based on the detailed 2-D nonlinear dynamics relaxes this constraint.
- The inefficiency of the control action in the nonlinear and saturated regimes highlight the importance of an early control action on small magnetic islands.
- However a broader ECCD causes a rapid fall of the control efficiency, due to the fact that a relevant fraction of the injected current falls outside the separatrices, even in presence of small magnetic island reduction.
- A more detailed analysis of the dependence of the efficiency on the beam amplitude and width is required as long as a study on the modulation of the beam.
- An extension to the low collisional regimes of the idea at the basis of ECCD could improve our understanding of the problem.