Two-stream Instability in Electron Accelerators

Nikolai Yampolsky
Los Alamos National Laboratory

Solved and Unsolved Problems in Plasma Physics
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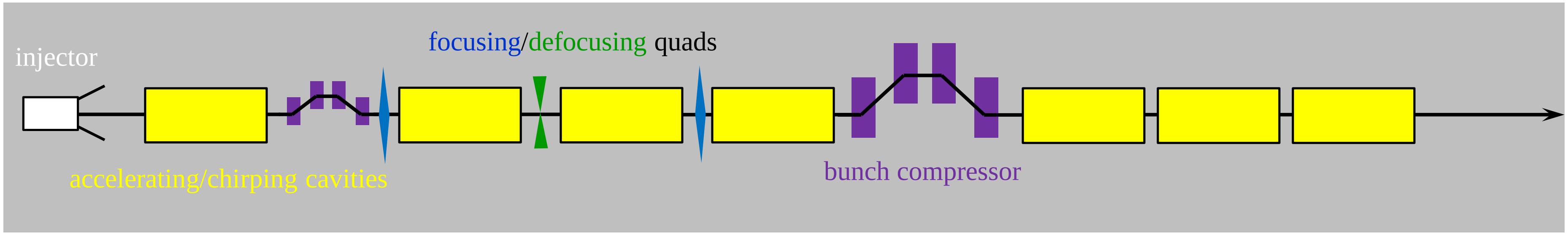
Accelerator layout



Charged particles are accelerated and controlled in external fields.

Bunch density decreases in the beam frame as n~1/γ. As a result, the external fields become larger than internal (e.g. charge separation) fields.

Accelerator elements:

- Bend (dipole) magnets
- Quadrupole magnets
- Solenoids
- RF cavities with various RF modes
- Vacuum drift spaces
- Undulators (for beam-radiation interactions)

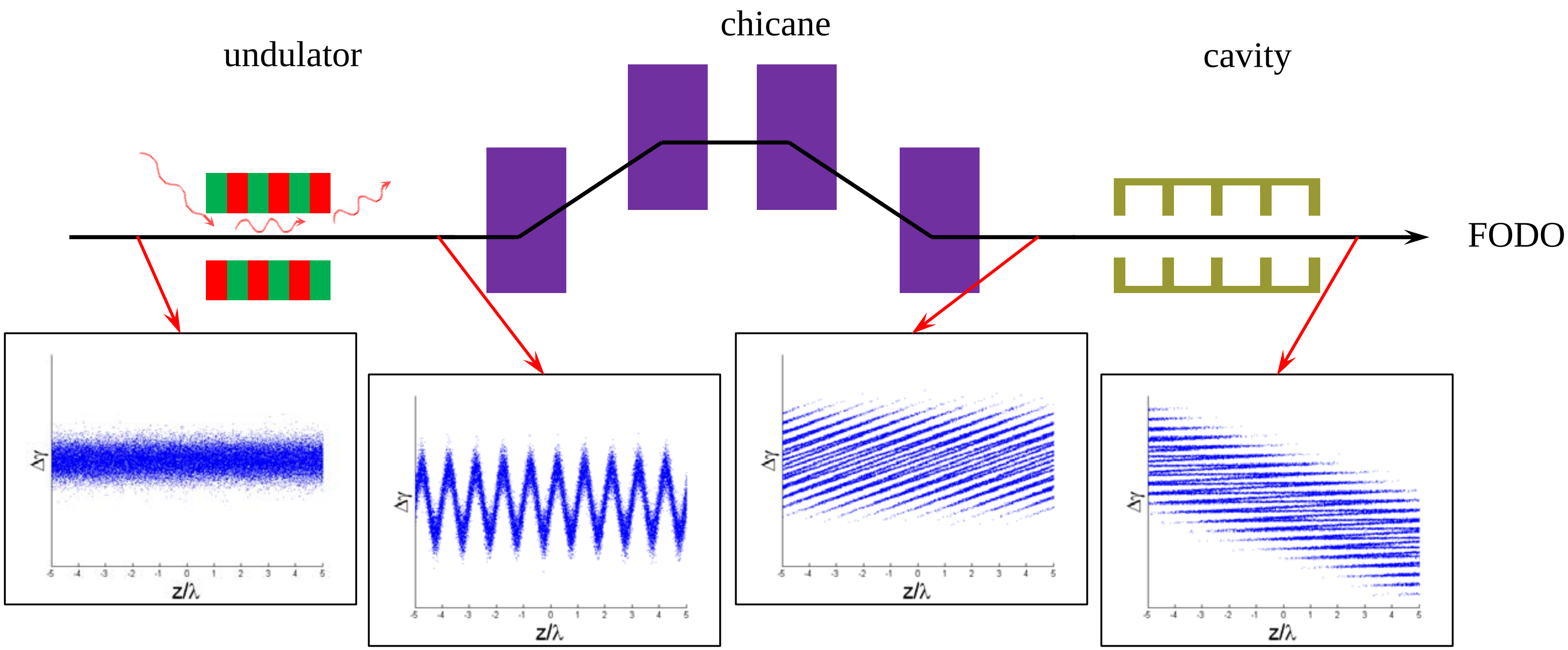
Complementary Plasma Physics in accelerators

Impressive success in Accelerator Physics is possible because of efficient control over the beam phase space:

- Focusing
- Compression
- Linearization of the phase space distribution
- Suppression collective effects

In accelerators it is possible to find a regime in which the phase space is carefully conditioned and then allow for collective effects to develop

Exploring two-stream instability in relativistic beams

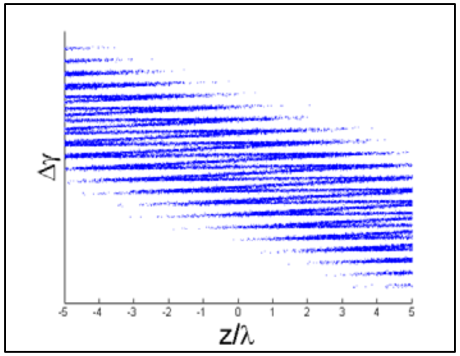


Final cavity is required to impose overall energy chirp to eliminate the residual chirp of each electron stream

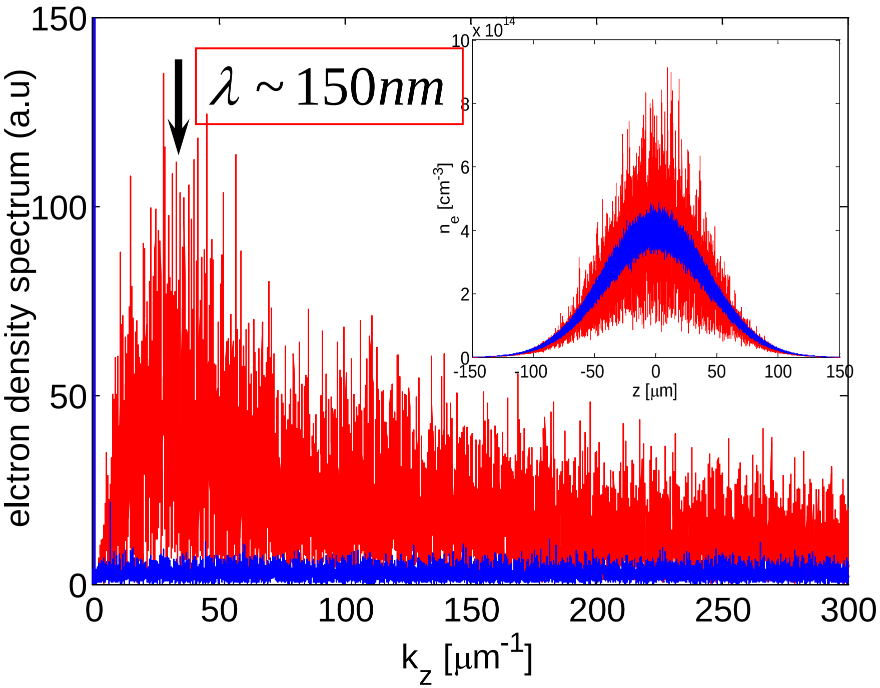
Numerical simulations

Electrostatic CPIC code was used to simulate N-stream instability in 1D.

Positrons with the same macroscopic distribution but different shot noise were added to suppress artificially large longitudinal space charge in 1D geometry



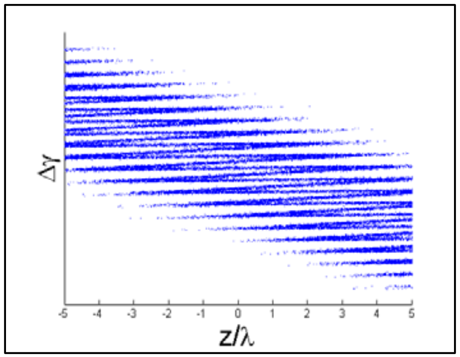
Initial electron distribution

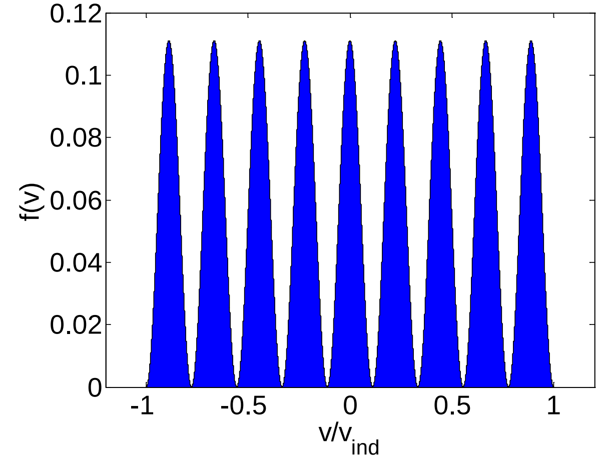


Parameters of the scheme were chosen to generate microbunching at 100nm.

Large density modulation is observed after 50m of vacuum drift

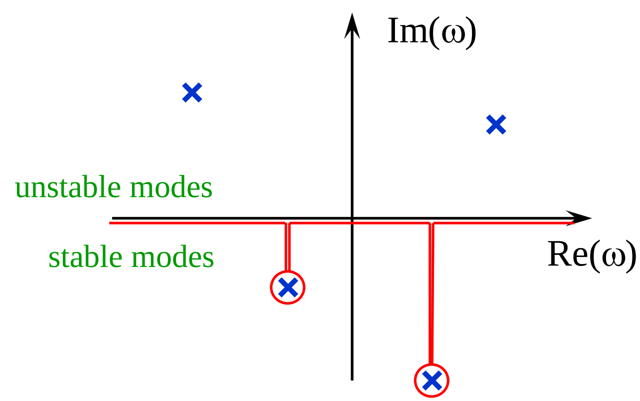
Treating electron beam as plasma



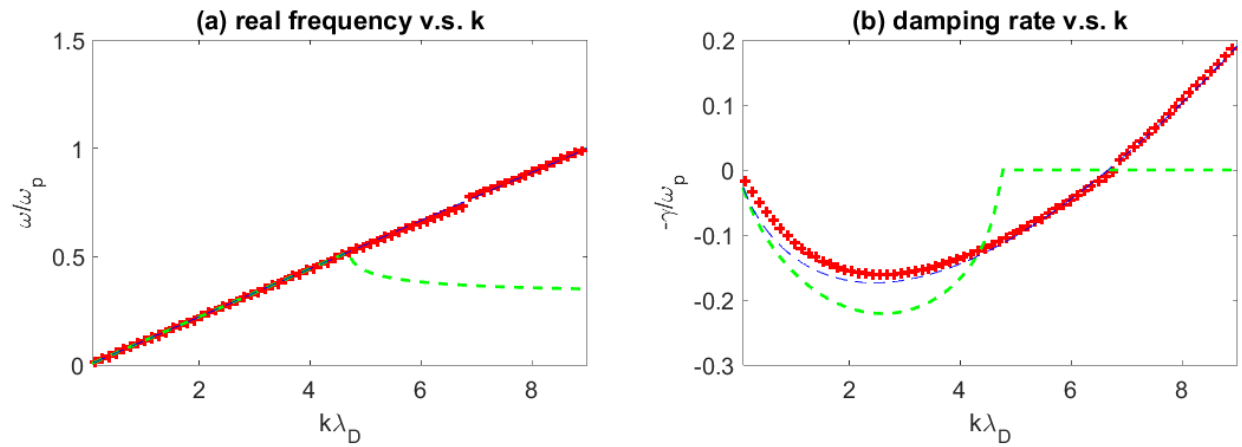


$$f(V)=\frac{1+\cos(\pi N_{bands}V/\Delta V_{ind})}{2\Delta V_{ind}},\quad V\in[-V_{ind},V_{ind}]$$

$$D(\omega,k)=1+\frac{\omega_p^2}{k}\int\frac{\partial_V f(V)}{\omega-kV}dV=0$$



Dispersion relation



Numerical dispersion
Cold fluid dispersion
Warm kinetic dispersion

$$\frac{\mathrm{Im}(\omega)}{\omega_p} = \frac{\Delta V_{ind} k}{\pi N_{bands}\omega_p} \ln\left(2N_{bands}\left[\frac{\Delta V_{ind} k}{\pi N_{bands}\omega_p}\right]^2\right)$$

$$\frac{\mathrm{Re}(\omega)}{\omega_p} = \frac{\Delta V_{ind} k}{N_{bands}\omega_p}$$

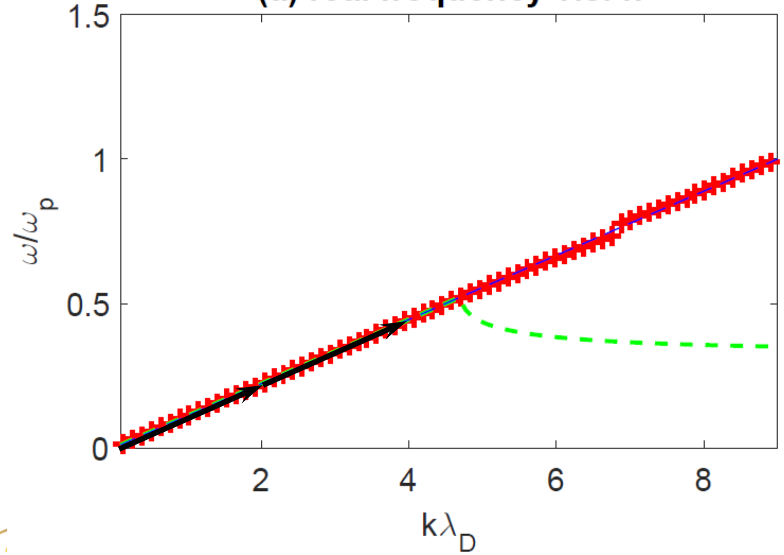
The frequency has both real (previously unaccounted for) and imaginary part

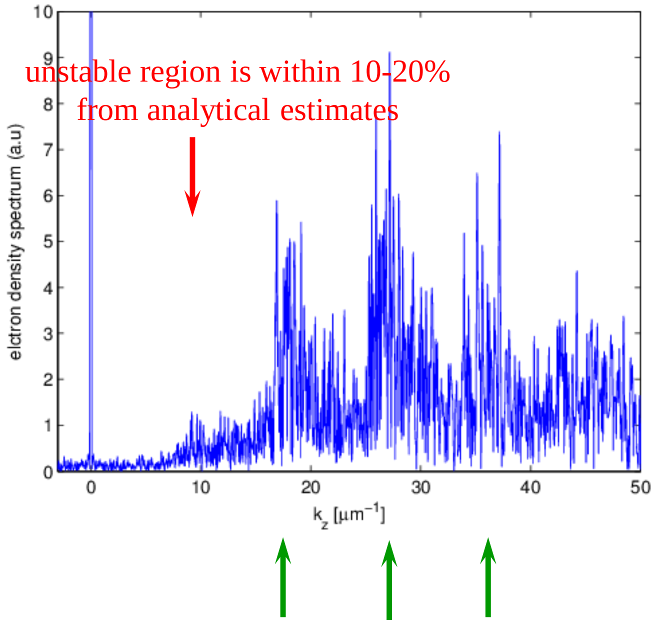
The dispersion looks like unstable sound wave
Re(ω)~k
Im(ω)<0

Checking with PIC simulations

Growth of plasma waves at small wavelengths can be explained by two-plasmon coupling

$$\omega_0 + \omega_0 = \omega_1 = 2\omega_0$$
$$k_0 + k_0 = k_1 = 2k_0$$





Additional peaks are at harmonics of the fundamental

Instability growth rate (lab frame)

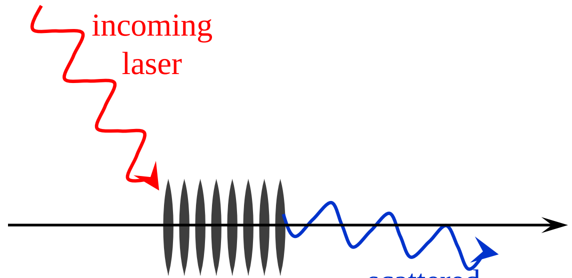
plasma density in the lab frame,
i.e. plasma frequency

$$\lambda_{\max} = \frac{0.12}{\gamma^{3/2}} \sqrt{\frac{r_\perp^2[100\mu m]\tau[ps]}{Q[nC]} \frac{\Delta E_{ind}[keV]}{N_{bands}}}[mm], L_g = \frac{c}{-\mathrm{Im}(\omega)} = 0.24\gamma^{3/2}\sqrt{N_{bands}\frac{r_\perp^2[100\mu m]\tau[ps]}{Q[nC]}}[mm],$$

Lorentz transform of time and space between the lab and beam frames;
(longitudinal mass in the lab frame scales as γ^3)

<u>NLCTA (at SLAC) parameters:</u>
γ=240, Q=0.3nC, $r_\perp$=100μm, τ=0.5ps, ΔE_{ind}=10keV, N_{bands}=10
the fastest growing mode has 60nm wavelength and 3.6m growth length

Application to Compton source



Microbunched beam can be considered as the density wave with dispersion relation

$$\omega=\beta c\cdot k$$

Compton scattering can be considered then as conventional 3-wave interaction

$$B \propto |b_k|^2$$

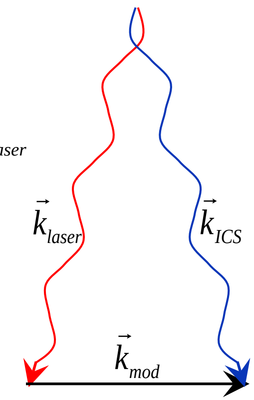
10³-10⁴ increase over unbunched beam

Conventional resonant conditions between coupled waves should be satisfied which makes ICS similar to Raman scattering

Beam frame resonance condition

$$\omega_{ICS} = \omega_{\mathrm{mod}} + \omega_{laser} \approx \omega_{laser}$$

$$\vec{k}_{ICS} = \vec{k}_{\mathrm{mod}} + \vec{k}_{laser}$$



Lab frame resonance condition

$$\boxed{\frac{1}{\lambda_{ICS}} = \frac{1}{\lambda_{laser}} + \frac{1}{\lambda_{mod}} \approx \frac{1}{\lambda_{mod}}}$$

Thank you!

Accelerator Physics

Vlasov equation in Beam Physics

$$\partial_t f + \nabla_{\vec{\varsigma}} f \cdot J \nabla_{\vec{\varsigma}} H = 0$$

$$\vec{\varsigma} = (x, p_x, y, p_y, \delta z, \delta p_z)$$

$$J = diag\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\right)$$

Characteristic (Newton) equation

$$\frac{d\vec{\varsigma}}{dt} = J \nabla_{\vec{\varsigma}} H \cong J \mathcal{H} \vec{\varsigma}$$

$$\vec{\varsigma}(t) = R \vec{\varsigma}(t = 0)$$

Electron coordinates in 6D phase space change linearly under linear forces applied